Tutorial 2 - Part 1 Solution

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Note:

$$\lambda = \frac{hc}{E(\text{in J})} = \frac{12400}{E(\text{in eV})}\,\text{Å} = \frac{1240}{E(\text{in eV})}\,\text{nm}$$

$$\lambda_{db} = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{2mEc^2}} = \frac{12400}{\sqrt{2\,(\text{Rest mass in eV})\,(\text{Energy in eV})}}\,\text{Å}$$

$E \downarrow$ KE of particle

1. Calculate the wavelength of the matter waves associated with the following:

   (a) A 2000 kg car moving with a speed of 100 km/h.

   (b) A 0.28 kg cricket ball moving with a speed of 40 m/s.

   (c) An electron moving with a speed of 10^7 m/s. $c = 3\times10^8$

   Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

Solⁿ (a) $p = mv$ $\lambda = \frac{h}{p}$

(b) - - -

(c) $v \simeq c \Rightarrow p \neq mv$ (relativistic formula to be used)

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} \simeq mv \qquad \beta = \frac{v}{c} = \frac{10^7}{3\times10^8} = \frac{1}{30}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\frac{mcv}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{12400}{\frac{mc^2\,\beta}{\sqrt{1-\beta^2}}}\,\text{Å} = \frac{12400\left(\sqrt{1-1/900}\right)}{(511\times10^3)(1/30)}$$

$$= 727.57\,\text{Å}$$

728.8 Å

③ 3. Calculate the de-Broglie wavelength (in nm) for a photon, an electron and a neutron each with an energy of 5 keV (for electron and neutron, the energy refers to non-relativistic kinetic energy). Take $m_e = 500$ keV/c² and $m_n = 1000$ MeV/c².

Solⁿ (i) For photon the wavelength of corresponding light is the de brogile wavelength

$$\lambda_{db} = \frac{h}{p} = \frac{h}{E/c} = \frac{hc}{E} = \lambda \qquad \left\{E = \frac{hc}{\lambda}\right\}$$

(ii) For e^-

$$\lambda_{db} = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2(mc^2)E}} = \frac{12400}{\sqrt{2\,(500\times10^3)\,(5\times10^3)}}\,\text{Å} = \frac{12400}{5\times10^4\sqrt{2}}\,\text{Å}$$

$$= 0.175\,\text{Å}$$

$$= 17.53\ \text{pm}$$

(iii) For n

$$\lambda_{db} = \frac{12400}{\sqrt{2(10^9)\,(5\times10^3)}}\,\text{Å} = \frac{12400}{\sqrt{10}\times10^6}\,\text{Å} = 0.39\ \text{pm}$$

(4)

4. *Thermal kinetic energy of a hydrogen atom is $\sim k_B T$ and the radius is $\sim r_1$ (= 0.53 Å, radius of the $n = 1$ Bohr orbit). Find the temperature at which its de Broglie wavelength has a value of $2r_1$. Take the mass of the hydrogen atom to be that of a proton.

Sol^n

$$\lambda_{db} = 2r_1 = 1.06 \times 10^{-10} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (1.67 \times 10^{-27}\ \text{kg})(E)}}$$

$$E = \left(\frac{6.63}{1.06}\right)^2 \times \frac{10^{-48}}{2 \times 1.67 \times 10^{-27}} = 11.71 \times 10^{-19}\ \text{J} = k_B T = \frac{R \cdot T}{N_A}$$

$$T = \frac{11.71 \times 10^{-21} \times 6.022 \times 10^{23}}{8.314}$$

$$T = 8.481 \times 10^{2}\ \text{K} \simeq 850\ \text{K}$$