Tutorial 2 - Part 2 Solution

19 December 2021 18:11

1. * Buckminsterfullerene are soccer-like balls (called buckyballs) made up of 60 carbon atoms (C_{60}). A double slit experiment is performed using these buckyballs travelling at a velocity of 100 m/sec (slit width = 150 nm and the separation between the slits and the screen, $D = 1.25$ m from the slits).

   (a) Find the de Broglie wavelength of the buckyball.

   (b) Find the distance between the maxima of the resultant interference pattern. Treat the buck balls as point like objects.

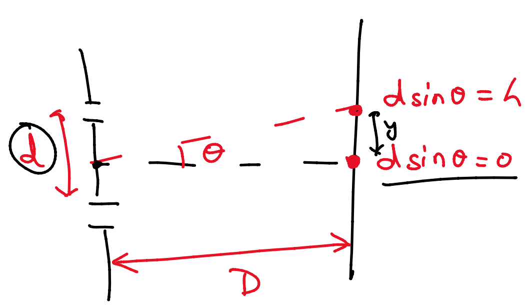
   (c) The size of the buckyballs is ~10 Å. How does the size of the ball compare with the distance between the neighboring maxima of the interference patterns? Is the size of C_{60} likely to affect the visibility of the interference fringes? Find the initial velocity of C_{60} for which the interference fringes start to become difficult to detect?

Sol^n

(a) $$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{60\times\frac{12}{6.022\times10^{23}}\times10^{-3}\times100} = \frac{6.63\times6.022}{60\times12}\times10^{-10}\ \text{m}$$

$$= 5.54\times10^{-12}\ \text{m}$$

(b)



$$d\sin\theta \simeq d\tan\theta = \frac{dy}{D} = \lambda$$

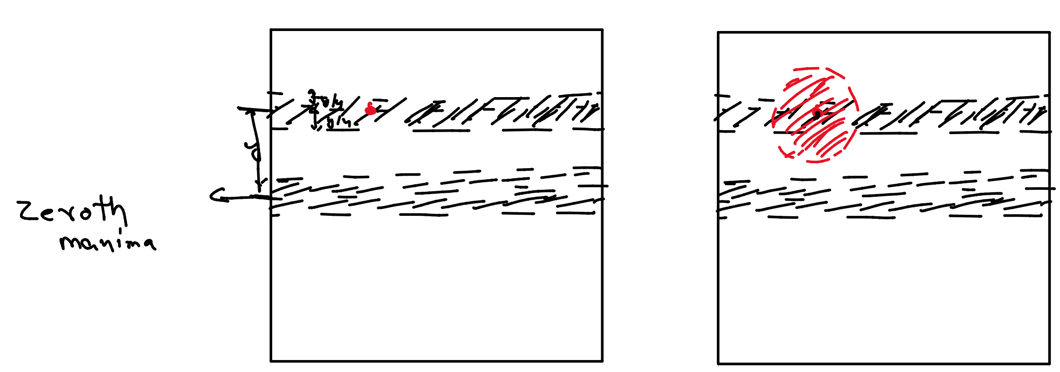
$$y = \frac{D\lambda}{d} = \frac{1.25\times5.54\times10^{-12}}{150\times10^{-9}}$$

$$= 4.616\times10^{-5}\ \text{m}$$

$$\beta_1 \simeq 46\ \mu m$$

(c) $\beta_1 = 461600$ Å $\gg$ 10 Å

No in this case, the size won't affect visibility



If the fringe width β becomes comparable to diameter (R_f) of fullerene then the maximas would start overlapping.

$$\beta = 10\ \text{Å} = \frac{D\lambda}{d} = \frac{1.25\cdot\lambda}{150\times10^{-9}} \Rightarrow \lambda = \frac{150\times10^{-18}}{1.25} = 1.2\times10^{-16}\ \text{m}$$

$$\Rightarrow \frac{h}{p} = 1.2\times10^{-16} \Rightarrow p = 5.5\times10^{-18}$$

$$\Rightarrow 60\times\frac{12}{6.022\times10^{23}}\times10^{-3}\times v = 5.5\times10^{-18}$$

$$\Rightarrow \boxed{v = 4.6\times10^{6}\ \text{m/s}}$$

⚠ If velocity comes out to be comparable to light ($\sim 0.1c$) then relativistic momentum should be used.

$$\frac{mv}{\sqrt{1-v^2/c^2}} = P$$

(2) 2. Consider two plane waves, one with a wave vector, $\vec{k}_1 = (2\pi/\lambda)(\vec{x}+\vec{y}+\vec{z})$, and the other with $\vec{k}_2 = (2\pi/\lambda)\vec{z}$. For $\lambda = 500$ nm, (a) find the resultant wave due to the interference of these two waves, (b) calculate the intensity and (c) analyze the interference pattern in the xy-plane, i.e. the condition for maxima and minima.

Sol^n Assuming the waves have same frequency ω. wave eqns can be written as

$$\psi_1(r,t) = A e^{i\left(\frac{2\pi}{\lambda}(x+y+z) - \omega t\right)}$$

$$\psi_2(r,t) = A e^{i\left(\frac{2\pi}{\lambda}(z) - \omega t\right)}$$

(a) $$\psi(r,t) = \psi_1(r,t) + \psi_2(r,t) = A\left[e^{i\left(\frac{2\pi}{\lambda}(x+y+z)-\omega t\right)} + e^{i\left(\frac{2\pi}{\lambda}z - \omega t\right)}\right]$$

(b) $$I = |\psi(r,t)|^2 = \psi(r,t)\cdot\psi^*(r,t)$$

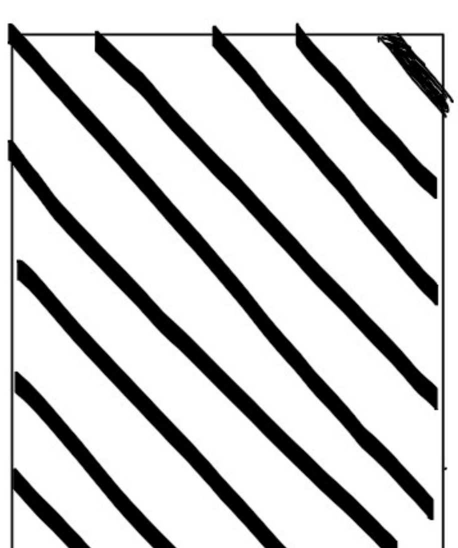
$$= |A|^2\left[e^{i(kx+ky+kz-\omega t)} + e^{i(kz-\omega t)}\right]\left[e^{-i(kx+ky+kz-\omega t)} + e^{-i(kz-\omega t)}\right]$$

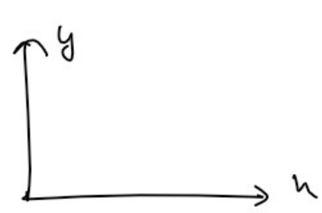
$$= |A|^2\left[1+1+2\cos(kx+ky)\right]$$

$$= 4|A|^2\cos^2\left(\frac{kx+ky}{2}\right)$$

(c) $\underline{k(x+y)} = (2n\pi)$ maxima $\qquad k(x+y) = \frac{(2n+1)\pi}{2}$ minima

$(x+y) = n\lambda \rightarrow$ maxima $\qquad x+y = \left(n+\frac{1}{2}\right)\lambda$ minima





(4) 4. *In a double-slit experiment with a source of monoenergetic electrons, detectors are placed along a vertical screen parallel to the y-axis to monitor the diffraction pattern of the electrons emitted from the two slits. When only one slit is open, the amplitude of the

$$\psi(y,t) = \underline{\psi_1(y,t)} + \underline{\psi_2(y,t)}$$

(4)

4. *In a double-slit experiment with a source of monoenergetic electrons, detectors are placed along a vertical screen parallel to the y-axis to monitor the diffraction pattern of the electrons emitted from the two slits. When only one slit is open, the amplitude of the electrons detected on the screen is $\psi_1(y,t) = A_1 e^{-i(ky-\omega t)}/\sqrt{1+y^2}$, and when only the other is open the amplitude is $\psi_2(y,t) = A_2 e^{-i(ky+\pi y-\omega t)}/\sqrt{1+y^2}$, where A_1 and A_2 are normalization constants. Calculate the intensity detected on the screen when
(a) both slits are open and a light source is used to determine which of the slits the electron went through and
(b) both slits are open and no light source is used.
(c) Plot the intensity registered on the screen as a function of y for cases (a) and (b).

$$\psi(y,t) = \psi_1(y,t) + \psi_2(y,t)$$

$$I_1 = |\psi_1(y,t)|^2 = \psi_1(y,t)\cdot\psi_1^*(y,t)$$

$$I_2 =$$

$$I = |\psi_1(y,t)+\psi_2(y,t)|^2 = (\psi_1(y,t)+\psi_2(y,t))\cdot(\psi_1^*(y,t)+\psi_2^*(y,t))$$

Sol^n (a) Normalization Const.

$$\int_{-\infty}^{\infty} \psi_1(y,t)\cdot\psi_1^*(y,t)\cdot dy = 1$$

$$\int_{-\infty}^{\infty} |\psi_1(y,t)|^2\cdot dy = 1$$

ψ_1 is normalised

$$\int_{-\infty}^{\infty} \frac{|A_1|^2}{(1+y^2)}\cdot dy = 1 \Rightarrow |A_1|^2\cdot\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right) = 1$$

$$|A_1| = \frac{1}{\sqrt{\pi}}$$

Similarly $|A_2| = \frac{1}{\sqrt{\pi}}$

Assuming A_1, A_2 to be +ve reals $\left(A_1, A_2 = \frac{1}{\sqrt{\pi}}\right)$

$$I = I_1 + I_2$$

Sol^n (a) $I = |\psi_1|^2 + |\psi_2|^2$

(When we try to observe electrons, interference pattern destroys & intensities simply add up)

$$I = \frac{1}{\pi(1+y^2)} + \frac{1}{\pi(1+y^2)}$$

$$= \frac{2}{\pi(1+y^2)}$$

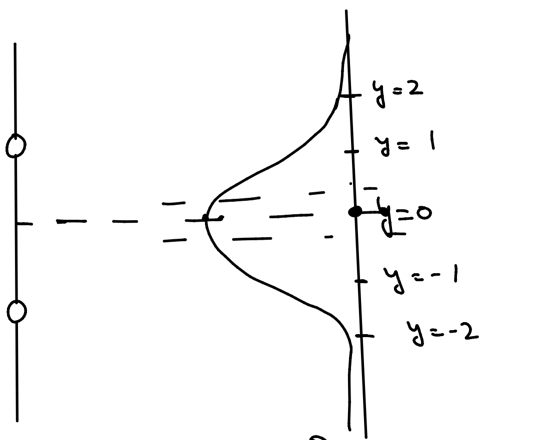
(b)

$$\psi = \psi_1(y,t) + \psi_2(y,t)$$

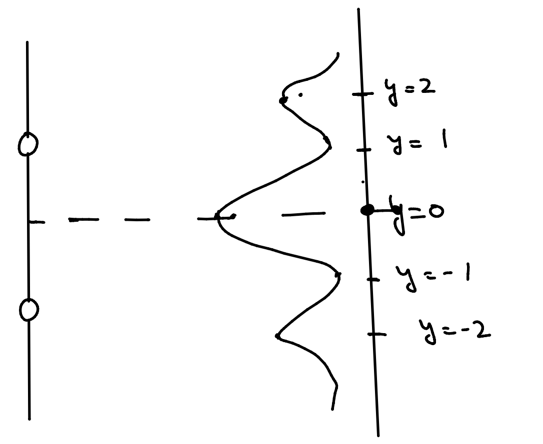
$$= \frac{A_1 e^{-i(ky-\omega t)}}{\sqrt{1+y^2}} + \frac{A_2 e^{-i(ky+\pi y-\omega t)}}{\sqrt{1+y^2}}$$

$$I = \psi\cdot\psi^* = \frac{|A_1^2|}{(1+y^2)} + \frac{|A_2^2|}{1+y^2} + \frac{(A_1 A_2^* e^{i\pi y} + A_1^* A_2 e^{-i\pi y})}{(1+y^2)}$$

$$= \frac{A_1^2}{(1+y^2)} + \frac{A_2^2}{(1+y^2)} + \frac{2A_1A_2\cos(\pi y)}{(1+y^2)} = \frac{4\cos^2(\pi y/2)}{\pi(1+y^2)}$$



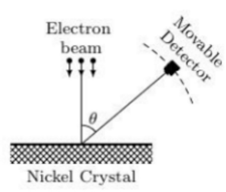
(a)



(b)

(5)

5. *In a Davisson-Germer experiment, electrons having energy of 54eV were bombarded normally over copper crystal. The diffracted beam was recorded using a detector and when the intensity of the diffracted electrons was plotted against the angle with the normal of the surface and the 1st maxima was observed at an angle of $\theta = 35°$.



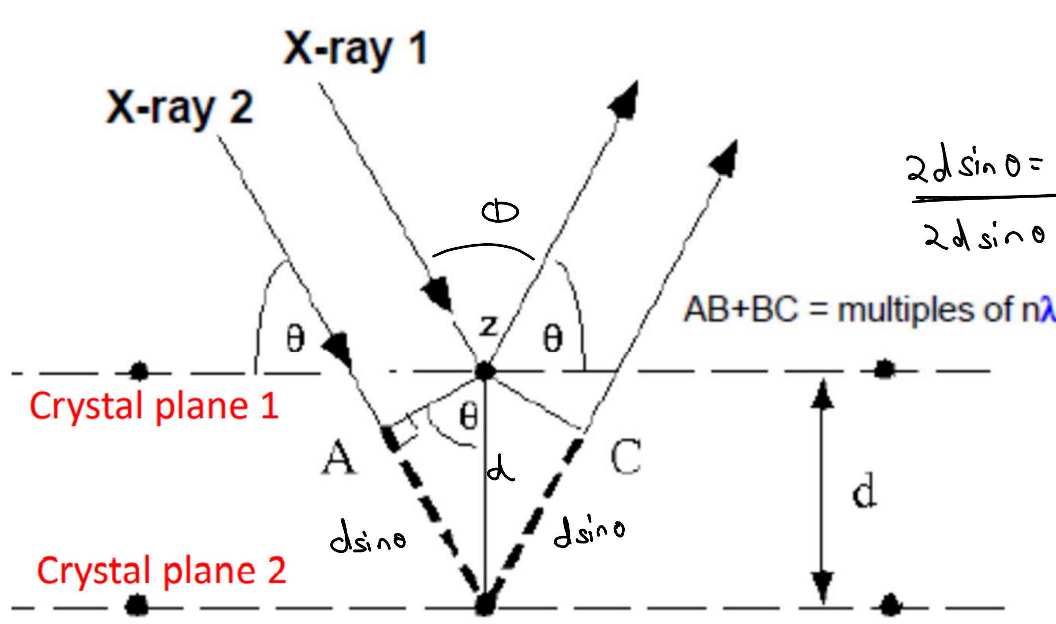
(a) Calculate the spacing between the atoms on the copper surface.

(b) What other angles are possible for a maxima?

(c) If the energy of incident electrons were increased by 3 times. Find the location of first maxima.

(d) How many more intensity peaks (maxima's) will be observed as the angle is further increased?

Sol^n



$2d\sin\theta = n\lambda \rightarrow$ Const. inter

$2d\sin\theta = \left(n+\frac{1}{2}\right)\lambda \rightarrow$ dest.

For given question $\quad \theta = 90° - \frac{35°}{2} = 72.5°$

(a) $2d\sin\theta = n\lambda = \lambda \quad$ ($n=1 \rightarrow$ first maxima)

$$d = \frac{\lambda}{2\sin(72.5°)}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times54\times1.6\times10^{-19}}} = \frac{6.63\times10^{-34}}{3.96\times10^{-24}}$$

$$= 1.67\times10^{-10}\ m$$

$$\boxed{d = 0.876\times10^{-10}\ m = 87.6\ pm}$$

(b) No other angle possible for the maxima.

$(\theta < 90^\circ)$ $\quad 2d\sin(72.5^\circ) = \lambda$

$2d \sin\theta' = 2\lambda$

No possible value of θ'

(c) $K' = K + 3K = 4K$ $\qquad \lambda \propto \frac{1}{\sqrt{K}} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$

$\lambda' = \lambda/2$

$2d\sin\theta = n\lambda' = \lambda'$ (first maxima $\rightarrow n = 1$)

$2d\sin\theta = \frac{\lambda}{2}$

$\frac{\sin 72.5^\circ}{\sin\theta} = 2 \Rightarrow \boxed{\theta = 28.48^\circ} \Rightarrow \phi = 2(90-\theta) = 123^\circ$

(d) $2d\sin\theta = n\lambda'$

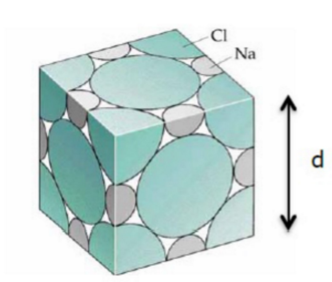
$n = 2$

$2d\sin\theta = 2\lambda' = \lambda$

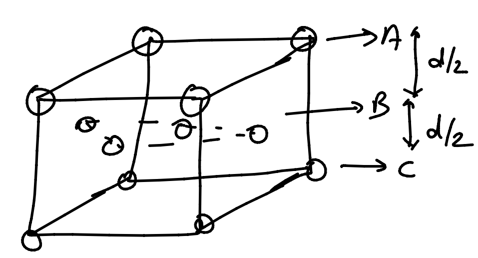
$\theta = 72.5^\circ \Rightarrow \boxed{\phi = 35^\circ}$

6

6. Sodium Chloride (NaCl) crystal is made up of cubes of edge length d, as shown in the figure. Each cube contains a full Na ion at its body center, which is not shown in the figure. In a Davisson-Germer experiment, performed using electrons of kinetic energy 40 eV, the NaCl crystal gives a first order ($n = 1$) diffraction peak at 20.11°.



fcc



(a) Compute d

(b) Compute the number of NaCl molecules in the given cube.

(c) Given the density and the molecular weight of NaCl to be 2.17 g/cm³ and 58.44 g/mol, respectively, compute Avogadro's number.

Sol^n (a) Assuming that the angle 20.11° is the angle b/w incident beam & surface

$2\left(\frac{d}{2}\right)\sin\theta = n\lambda = \lambda$

$d = \frac{\lambda}{\sin\theta} = \frac{h}{\sqrt{2mK}\,\sin\theta} = \frac{6.63\times10^{-34}}{2\times 0.343 \times \sqrt{2\times 9.1\times10^{-31} \times 40 \times 1.6\times10^{-19}}}$

$= \frac{9.66 \times 10^{-9}}{34.129}\ m = \boxed{0.576\ nm}$

(b) CCP structure of NaCl $\qquad Cl^- \rightarrow \frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$

$1\times 12 = 4$

(b) CCP structure of

$Na^+ \rightarrow 1 + \frac{1}{4} \times 12 = 4$ $Cl^- \rightarrow \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

4 NaCl molecules

(c) Wt. of 1 mole crystal = $4 \times 58.44\,g = (N_A)(d^3)(\rho)$

$$N_A = \frac{4 \times 58.44\,g}{(57.6 \times 10^{-9}\,cm)^3 \times (2.17\,g/cm^3)}$$

$$= 5.94 \times 10^{23} \simeq 6 \times 10^{23}$$