Tutorial 11 Solution

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4. * Consider a particle confined to a 3D harmonic oscillator potential, $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + 4z^2)$

(a) Calculate the ground state energy of the particle.

(b) What is the degeneracy of the state with energy, $E = 7\hbar\omega$?

$\frac{1}{2} m \omega^2 q^2 \rightarrow \frac{1}{2} m \omega^2 (4z^2)$, $(2\omega)^2 z^2$

$E_n = (n_x + \frac{1}{2})\hbar\omega$

Sol^n $V(x,y,z) = \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m \omega^2 y^2 + \frac{1}{2} m (2\omega)^2 z^2$

$\rightarrow (n_y + \frac{1}{2})\hbar\omega$ $\quad \rightarrow (n_z + \frac{1}{2})(\hbar \cdot 2\omega)$

(a) $\frac{\hbar\omega}{2} + \frac{\hbar\omega}{2} + \hbar\omega = 2\hbar\omega$

(b) $7\hbar\omega = \hbar\omega\left(n_x + \frac{1}{2}\right) + \hbar\omega\left(n_y + \frac{1}{2}\right) + \hbar(2\omega)\left(n_z + \frac{1}{2}\right)$

$5 = n_x + n_y + 2n_z$

n_x	n_y	n_z
5	0	0
4	1	0
3	2	0
2	3	0
1	4	0
0	5	0
3	0	1
2	1	1
1	2	1
0	3	1
1	0	2
0	1	2

$\boxed{\text{degeneracy} \rightarrow 12}$

8. * Consider a system of five particles trapped in a 1-D harmonic oscillator potential.

(a) What are the microstates of the ground state of this system for classical particles, identical Bosons and identical spin half Fermions.

(distinct classical Particles)

(b) Suppose that the system is excited and has one unit of energy ($\hbar\omega$) above the corresponding ground state energy in each of the three cases. Calculate the number of microstates for each of the three cases.

(c) Suppose that the temperature of this system is low, so that the total energy is low (but above the ground state), describe in a couple of sentences, the difference in the behavior of the system of identical bosons from that of the system of classical particles.

Sol: $\sigma = \{n_1, n_2, n_3, n_4, n_5\}$ $\qquad E_i = (n_i + \frac{1}{2})\hbar\omega$

where $n_1, n_2, n_3, n_4, n_5 \in \{0, 1, 2, \ldots\}$

Ground state $\rightarrow$ lowest energy state

Distinguishable particles $\rightarrow$ $\sigma = \{0,0,0,0,0\} \rightarrow 1$

Bosons $\qquad \sigma = \{0,0,0,0,0\} \rightarrow 1$

Identical spin-half fermions $\rightarrow$ $\left[\sigma_1 = \{\underset{\uparrow}{0}, \underset{\downarrow}{0}, \underset{\uparrow}{1}, \underset{\downarrow}{1}, \underset{\uparrow}{2}\},\ \sigma_2 = \{0, 0, 1, 1, \underset{\downarrow}{2}\} \right]$

$+\frac{1}{2} \quad -\frac{1}{2}$

(b) Distinguishable particles $\rightarrow$ $\sigma_1 = \{1,0,0,0,0\}$

$\sigma_2 = \{0,1,0,0,0\}$ $\qquad \rightarrow$ 5 microstates

$\vdots$

$\sigma_5 = \{0,0,0,0,1\}$

Identical bosons $\qquad \sigma_1 = \{1, \underbrace{0,0,0,0}_{\text{identical}}\}$ $\rightarrow$ 1 micro state

identical so only 1 micro state

Fermions

$G_1 = \{0, 0, 1, 1, 3\}$ 1

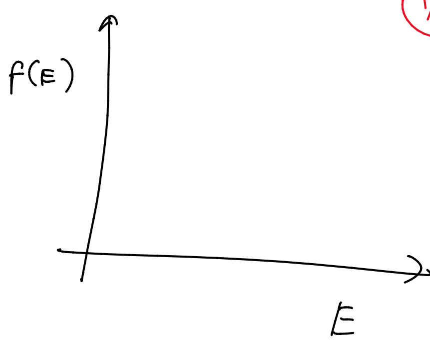
$G_2 = \{0, 0, 1, 2, 2\}$

$G_3 = \{0, 0, 1, 2, 2\}$

$G_4 = \{0, 0, 1, 1, 3\}$

4 microstates possible

(c) $-k_b \ln(W)$



2nd excited

$1/15 = f(E)_{\text{classical}} = e^{-E/kT} = \dfrac{1}{e^{E/kT}}$

2 (0,2,0,0,0)
(1,1,0,0,0)

$1/2 = f(E)_{\text{Boson}} = \dfrac{1}{-1+e^{E/kT}}$

15 microstate

10 (1,1,0,0,0)

5 (2,0,0,0,0)

$f(E)_{\text{fermion}} = \dfrac{1}{e^{E/kT}+1}$