Tutorial 4 Solutions HUP

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HUP $\rightarrow$ $\Delta y = \sqrt{\langle y^2\rangle - \langle y\rangle^2}$ (Similar to std. deviation)

$\langle x\rangle \rightarrow$ expectation value of x

a real measurable quantity

Usually $\langle y\rangle = 0$ so $\boxed{\Delta y = \sqrt{\langle y^2\rangle}}$

$\hookrightarrow$ eg. (If suppose we have an $e^{\ominus}$ in a box then it moves randomly in all directions to give $\langle p\rangle = 0$)

2. A lead nucleus has a radius 7×10^{-15} m. Consider a proton bound within nucleus. Using the uncertainty relation $\Delta p.\Delta r \geq \hbar/2$, estimate the root mean square speed of the proton, assuming it to be non-relativistic. (You can assume that the average value of p^2 is square of the uncertainty in momentum.)

Sol^n given: $\langle p^2\rangle = \Delta p^2$

$$\sqrt{\langle p^2\rangle} \cdot \Delta r \geq \hbar/2$$

$$m\sqrt{\langle v^2\rangle} \cdot \Delta r \geq \hbar/2 \implies \boxed{\sqrt{\langle v^2\rangle} \geq \left(\hbar/2m\right)\left(1/\Delta r\right)}$$

3. *For a non-relativistic electron, using the uncertainty relation $\Delta x \Delta p_x = \hbar/2$
(a) Derive the expression for the minimum kinetic energy of the electron localized in a region of size ' a '.
(b) If the uncertainty in the location of a particle is equal to its de Broglie wavelength, show that the uncertainty in the measurement of its velocity is same as the particle velocity. (upto a constant factor)
(c) Using the expression in (b), calculate the uncertainty in the velocity of an electron having energy 0.2keV
(d) An electron of energy 0.2keV is passed through a circular hole of radius 10^{-6} m. What is the uncertainty introduced in the angle of emergence in radians? (Given $\tan\theta \cong \theta$)

Sol^n (a) $\Delta x = a$ $\qquad \Delta p_x = \dfrac{\hbar}{2a}$ $\qquad \Delta p_x = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2}$

$$\langle K_x\rangle = \frac{\langle p_x^2\rangle}{2m} = \frac{((\Delta p_x)^2 + \langle p_x\rangle^2)}{2m} \qquad \langle p_x\rangle = 0$$

$$(K_x)_{min} = \langle K_x\rangle_{min} = \frac{(\Delta p_x)^2}{2m} = \frac{(\hbar/2a)^2}{2m} = \frac{\hbar^2}{8a^2 m}$$

(b) given $\Delta x = \dfrac{h}{p_x} = \lambda_b$

$$\frac{h}{p_x} \cdot \Delta p_x = \frac{\hbar}{2}$$

$$\frac{h \cdot \cancel{m} \Delta V_n}{\cancel{m} V_n} = \frac{\hbar}{2}$$

$$\Delta V_n = \frac{V_n}{4\pi}$$

(c) $V_n = \sqrt{\frac{2E}{m_{e\theta}}} = \sqrt{\frac{2(0.2\ KeV)}{(511\ KeV/c^2)}} = \sqrt{\frac{0.4}{511}}\ c = \underline{8.39 \times 10^6\ m/s}$

velocity of light

$\Delta V_n = \frac{V_n}{4\pi} = \underline{6.67 \times 10^5\ m/s}$

(d) $\Delta\theta = \frac{\Delta P_y}{P_n}$

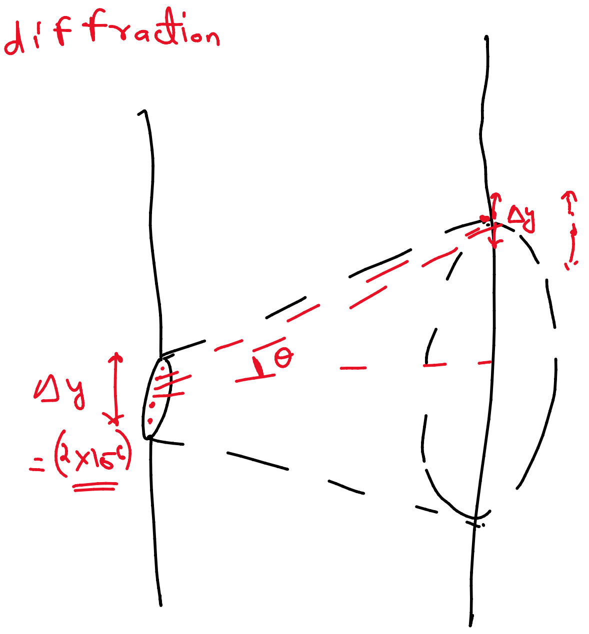
uncertainity arising due to diffraction from the slit

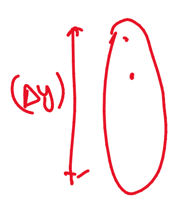
$(\Delta y = 10^{-6} m)$

$\Delta y \cdot \Delta P_y = \frac{\hbar}{2}$

$\left(\Delta P_y = \frac{\hbar}{2\Delta y}\right)$

$\Delta\theta = \frac{(\hbar/2\Delta y)}{\sqrt{2Em}}$





$\Delta y \cdot \Delta P_y = \frac{\hbar}{2}$