Tutorial 10 Supplement

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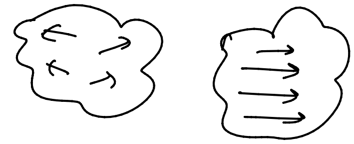
→ Just similar to Polarization $\vec{P}$ we have magnetization $\vec{M}$ (magnetic dipole moment/ volume)

$\vec{E}$



BOUND CURRENTS

$\vec{P} = \frac{\vec{p}}{V}$

$\vec{B}$

$\boxed{\vec{M} = \frac{\vec{m}}{V}}$

$J_b = \nabla \cdot \vec{P}$

$\sigma_b = \vec{P} \cdot \vec{n}$

Define: $\boxed{\begin{aligned} \boldsymbol{J}_b &= \nabla \times \boldsymbol{M} \\ \boldsymbol{K}_b &= \boldsymbol{M} \times \hat{\boldsymbol{n}} \end{aligned}}$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{J}_b(\boldsymbol{r}')}{\underbrace{\boldsymbol{r}-\boldsymbol{r}'}}d^3r' + \frac{\mu_0}{4\pi}\oint \frac{\boldsymbol{K}_b(\boldsymbol{r}')}{\underbrace{\boldsymbol{r}-\boldsymbol{r}'}}da'$$

$B(\vec{r})$

The magnetic vector potential (and hence the field) of a magnetized object is the same as would be produced by a volume current $\boldsymbol{J}_b = \nabla \times \boldsymbol{M}$ throughout the material, plus a surface current $\boldsymbol{K}_b = \boldsymbol{M} \times \hat{\boldsymbol{n}}$ on the boundary

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\left[\oint \frac{\sigma_b(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}da' + \int \frac{\rho_b(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3\boldsymbol{r}'\right]$$

$$\sigma_b \equiv \boldsymbol{P}.\hat{\boldsymbol{n}} \qquad \rho_b \equiv -\nabla.\boldsymbol{P}$$

1. A cylindrical magnet of length $2L$ and radius R has a uniform magnetization $\vec{M} = M_0\hat{k}$.

   (a) Find the volume current density $\vec{J}_b$ and the surface current density $\vec{K}_b$. [Ans. $\vec{J}_b = 0$, $\vec{K}_b = M_0\hat{\phi}$]

   (b) Find the magnetic field at a point P $(0, 0, z)$ where $|z| > L$. The origin of the coordinate system is fixed at the center of the cylinder. [Ans. $\vec{B} = \frac{\mu_0 K_b}{2}(\cos\theta_2 - \cos\theta_1)\hat{k}$, $\cos\theta_1 = (z-L)/\sqrt{(R^2+(z-L)^2)}$, $\cos\theta_1 = (z+2L)/\sqrt{(R^2+(z-L)^2)}$]

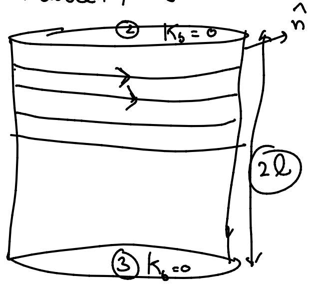
$(\vec{J}_p \ \& \ \vec{K}_p = 0)$

Sol^n (a) $\vec{J}_b = \nabla \times \vec{M} = 0$ ($\vec{M}$ is const.)

$\vec{K}_b = \vec{M} \times \vec{n} = M_0\hat{\phi}$ $(\hat{k} \times \hat{r} = \hat{\phi})$

↳ current / length

(b)



$\hat{n} = \hat{r}$

$\hat{M} = \hat{k}$

$\vec{K}_b$ ↓ Current/length $= M_0$

Can be considered to be similar to a solenoid

$$\vec{B}_{axis} = \frac{\mu_0 ni}{2}(\underbrace{\cos\theta_2 - \cos\theta_1}) = \frac{\mu_0 K_b}{2}(\cos\theta_2 - \cos\theta_1)$$

ni → Total current $(K_b l)$ = $\boxed{K_b}$

$n\,i$

no. of turns / length

$(1/2l)$

4. A region is occupied by an infinite slab of a permeable material of constant relative permeability $\mu_r = \mu/\mu_0 = 2.5$. The slab is infinite in the x and y directions, and is confined between $0 \le z \le 2$. Within the slab, the magnetic field (in Wb/m^2) is given by

$$\vec{B} = 10y\hat{\imath} - 5x\hat{\jmath}.$$

Determine $\vec{J}_f$, $\vec{J}_b$, $\vec{M}$, and $\vec{K}_b$. [Ans $\vec{J}_f = -\frac{6}{\mu_0}\hat{k}$, $\vec{J}_b = -\frac{9}{\mu_0}\hat{k}$, $\vec{M} = \frac{3}{5\mu_0}\left(10y\hat{\imath} - 5x\hat{\jmath}\right)$, $\vec{K}_b(z=0) = \frac{3}{5\mu_0}\left(10y\hat{\jmath} + 5x\hat{\imath}\right)$, $\vec{K}_b(z=2) = -\frac{3}{5\mu_0}\left(10y\hat{\jmath} + 5x\hat{\imath}\right)$

$\vec{P} \propto \vec{E}$

$\vec{P} = \alpha\vec{E}$

$\vec{M} = \alpha\vec{B}$

Sol

$\varepsilon_0\vec{E} - \vec{P} = \vec{D}$

$(\nabla\cdot\vec{D} = \rho_f)$

$\frac{\vec{B}}{\mu_0} - \vec{M} = \vec{H}$

$\nabla\times\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \boxed{\nabla\times\vec{H} = J_f}$

$\vec{H} = k\vec{M}$

$\nabla\times\vec{M} = \vec{J}_b$

$\vec{M}\times\hat{n} = \vec{K}_b$

$\vec{J}_b$, $\vec{K}_b$, $\vec{M}$

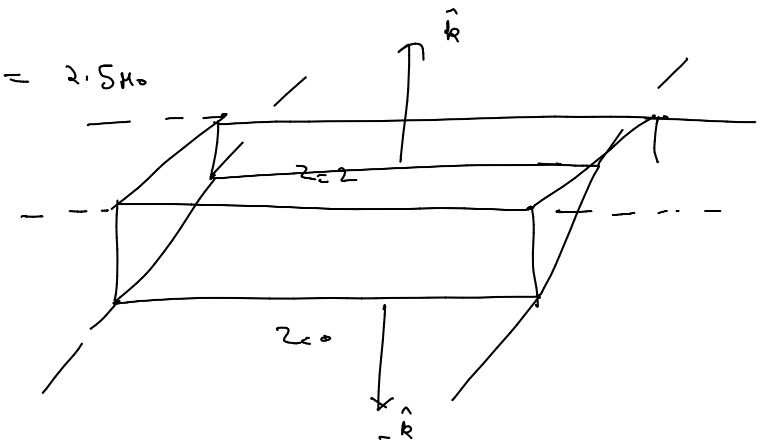
$\vec{M}$

$\frac{\vec{B}}{\mu_0} - \vec{M}$

$\vec{H} = \frac{\vec{B}}{\mu} = \frac{\vec{B}}{2.5\mu_0}$

$\mu = \mu_0\mu_r = 2.5\mu_0$

$$\boxed{\vec{M} = \frac{\vec{B}}{\mu_0} - \frac{\vec{B}}{2.5\mu_0} = \frac{3}{5}\frac{\vec{B}}{\mu_0}}$$



$\vec{J}_b = \nabla\times\vec{M}$

$\vec{K}_b = \vec{M}\times\hat{n}$