Tutorial 4 Solution

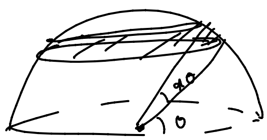
29 April 2022 12:34

1. * Consider a conducting sphere A which is initially uncharged. Another conducting sphere B is given a charge $+Q$, brought into contact with A and then moved far away. The charge on B is then increased to its original value $+Q$ and again brought into contact with A. Show that if this process is repeated many times, the charge on A will tend to the limit $\frac{Qq}{Q-q}$, where q is the charge acquired by A after its first contact with B.

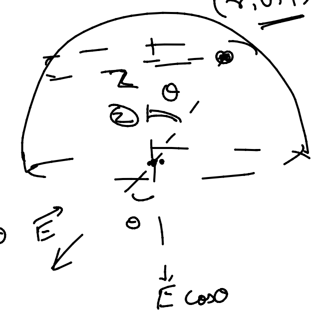
Sol^n

2. *A hemisphere of radius R has $z = 0$ as its equatorial plane and lies entirely in the region $z \geq 0$. The hemisphere has a uniform volume charge density of ρ. Determine the field at the centre of the hemisphere.

Sol^n



$\theta = 0$ to $\frac{\pi}{2}$



$z = r\cos\theta$

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho\, d\tau}{r^2}\cos\theta$$

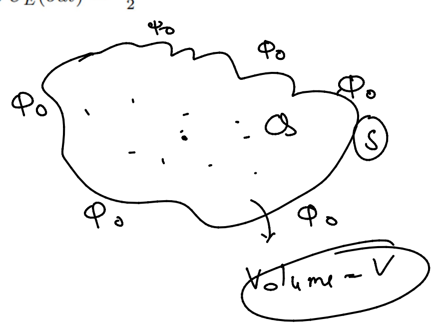
$$E_z = \int dE_z = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{R} \frac{1}{4\pi\varepsilon_0} \frac{\rho \cdot \cancel{r^2}\sin\theta\cos\theta\, dr\, d\theta\, d\phi}{\cancel{r^2}} \qquad \int \sin\theta$$

$$= \frac{\rho \cdot R}{4\pi\varepsilon_0} \times 2\pi \int_0^{\pi/2} \frac{1}{2}\sin 2\theta = \frac{\rho R}{4\varepsilon_0}\,\frac{1}{2} \times 2 = \frac{\rho R}{4\varepsilon_0}$$

$$-\cos 2\theta \Big|_0^{\pi/2} \qquad -\cos\pi + \cos 0 = 2$$

3. * The potential takes the constant value ϕ_0 on the closed surface S which bounds a volume V. The total charge inside V is Q. There is no charge anywhere else. Show that the electrostatic energy contained in the space outside of S is $U_E(out) = \frac{Q\phi_0}{2}$

Sol^n



$$E = \frac{1}{2}\underbrace{\iiint}_{\text{Whole space}} \rho V\, d\tau \equiv \frac{1}{2}\iiint_{V} \rho V\, d\tau$$

$\tau > V$

$\rho = 0 \rightarrow$ (no external charges

$$U_{Total} = \frac{1}{2}\iiint \rho \underset{\hookrightarrow \text{pot.}}{V}\, d\tau$$

i.e. over whole volume

$U_{\underline{Total}} = \frac{1}{2}\iiint_V \rho V \, d\tau$ ↳ pot.

Volume integral is over whole volume
surface integral would be over
the surface at ∞.

$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

Volume integral is over volume V
surface integral would be over
the surface S.

$$= \frac{\varepsilon_0}{2}\iiint_V (\nabla \cdot \vec{E}) V \, d\tau$$

$$\nabla \cdot (f\vec{A}) = \vec{A} \cdot (\nabla f) + f(\nabla \cdot \vec{A})$$

$$= \frac{\varepsilon_0}{2}\iiint_V \left[\nabla \cdot (V\vec{E}) - \vec{E} \cdot (\nabla V)\right] \cdot d\tau$$

$$\underbrace{\iiint_V (\nabla \cdot \vec{A}) \, d\tau = \iint (\vec{A} \cdot d\vec{s})}$$

↓

Divergence

S is boundary of volume is

$$U_{Total} = \frac{\varepsilon_0}{2}\iint_S V(\vec{E} \cdot d\vec{s}) + \frac{\varepsilon_0}{2}\iiint_V (\vec{E} \cdot \vec{E}) \, d\tau$$

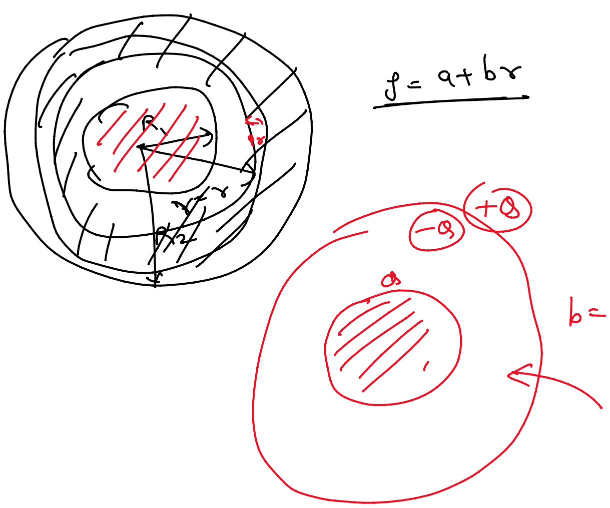
$$\underbrace{\qquad}_{\parallel} \; U_{\underline{inside}}$$

$$U_{total} - U_{inside} = \frac{\varepsilon_0}{2}\iint_S V(\vec{E} \cdot d\vec{s}) \qquad V \to \Phi_0$$

$$U_{outside} = \frac{\varepsilon_0 \Phi_0}{2}\iint_S (\vec{E} \cdot d\vec{s})$$

$$\boxed{U_{outside} = \frac{\varepsilon_0 \Phi_0 Q}{2} \cdot \frac{1}{\varepsilon_0} = \frac{Q\Phi_0}{2}}$$

(4)



$\rho = a + br$

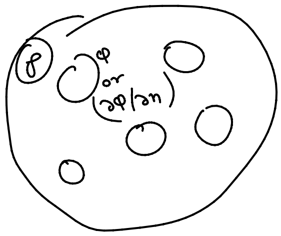
$b =$

$$dV = \frac{K \cdot dq}{r} = \frac{K(a+br)4\pi r^{\cancel{2}} \, dr}{\cancel{r}}$$

$$V = \int_{R_1}^{R_2} 4\pi K (ar + br^2) \, dr$$

$$V_{Centre} = 4\pi K \left[\frac{a}{2}(R_2^2 - R_1^2) + \frac{b}{3}(R_2^3 - R_1^3)\right]$$

(5)



Second Uniqueness Theorem: