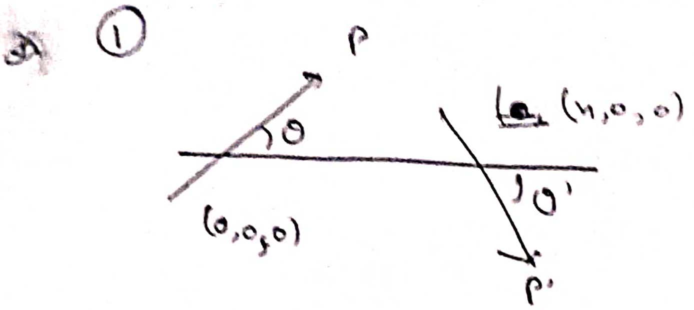
①



$\vec{E}(x,0,0) = \frac{2P\cos\theta \cdot K}{x^3}\hat{x} - \frac{KP\sin\theta}{x^3}\hat{y}$

Potential Energy of p'

$= -\vec{p}' \cdot \vec{E}$

$= -\left[\frac{2KPP'\cos\theta\cos\theta'}{x^3} + \frac{KPP'\sin\theta\sin\theta'}{x^3}\right]$

p' would be in a configuration where its PE is minimised. $\Rightarrow$ ~~$|\vec{p}' \cdot \vec{E}|$ is maxim~~ $\vec{p}' \cdot \vec{E}$ is maximum

$(c = \frac{KPP'}{x^3})$

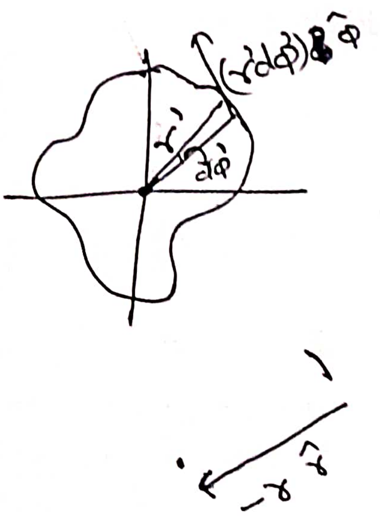
$f(\theta') = 2c\cos\theta\cos\theta' + c\sin\theta\sin\theta'$

$f'(\theta') = -2c\cos\theta\sin\theta' + c\sin\theta\cos\theta' = 0$

$\tan\theta' = \frac{\tan\theta}{2}$

$$\boxed{\theta' = \tan^{-1}\left(\frac{\tan\theta}{2}\right)}$$

② (a)



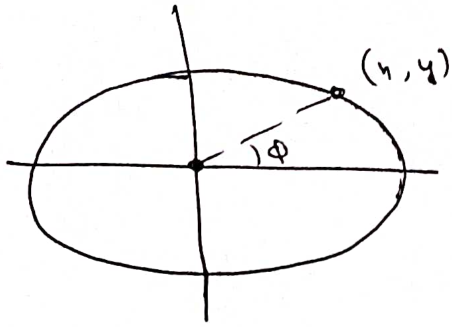
From Biot Savart's Law

$B = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$ r

$= \frac{\mu_0 I}{4\pi}\int_{wire} \frac{(r' d\phi' \hat{\phi}') \times (-r'\hat{r})}{|r'|^3}$

$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{d\phi'}{r(\phi')}\hat{z} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{d\phi}{r(\phi)}\hat{z}$

(b)



$$\tan\phi = \frac{y}{x}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{x^2\tan^2\phi}{b^2} = 1$$

$$x^2 = \frac{a^2b^2}{b^2 + a^2\tan^2\phi}$$

$$r^2 = x^2 + y^2 = x^2\sec^2\phi = \frac{a^2b^2\sec^2\phi}{b^2 + a^2\tan^2\phi} = \frac{a^2b^2}{b^2 + (a^2-b^2)\sin^2\phi}$$

$$= \frac{\cancel{a}^2}{1 \mp \cancel{\left(1 - \frac{a^2}{b^2}\right)}}$$

$$= \frac{a^2}{1 - \left(1 - \frac{a^2}{b^2}\right)\sin^2\phi}$$

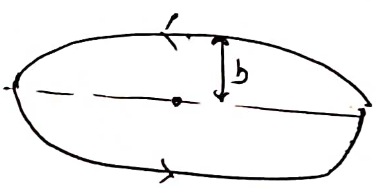
$$\vec{B} = \frac{\mu_0 I}{4\pi}\, 4 \int_0^{\pi/2} \frac{d\phi}{a} \sqrt{1 - \left(1 - \frac{a^2}{b^2}\right)\sin^2\phi}$$

$$\vec{B} = \frac{\mu_0 I}{\pi a} \int_0^{\pi/2} d\phi \sqrt{1 - \left(1 - \frac{a^2}{b^2}\right)\sin^2\phi}$$

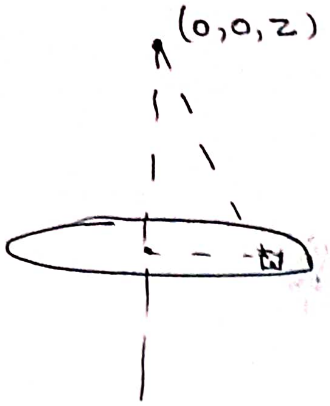
(i) $a = b \Rightarrow \vec{B} = \frac{\mu_0 I}{2a}$

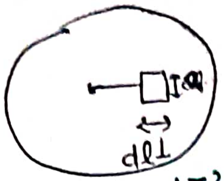
(ii) $a \to \infty$

$\frac{\mu_0 I}{\pi b}$ (double wire)



(c)





$dl_{\parallel} \cdot dl_{\perp} = da$

$\vec{I} = \vec{K}\, dl_{\perp}$

$\vec{r} = z\,\hat{z}$

$\vec{r}' = r'\,\hat{r} + \phi'\,\hat{\phi}$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{(\overbrace{\vec{K}\, dl'_{\perp}}^{dI'}) \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}\, dl'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{(K\,\hat{\phi}) \times (z\,\hat{z} - r\,\hat{r} - \phi'\,\hat{\phi})}{(r^2+z^2)^{3/2}}\, \boxed{r'\, dr'\, d\phi'}$$

$$= \frac{\mu_0}{4\pi}\int_0^R \int_0^{2\pi} \frac{(Kz\,\hat{r} + r'K\,\hat{z})}{((r')^2+z^2)^{3/2}}\, r'\, dr'\, d\phi'$$

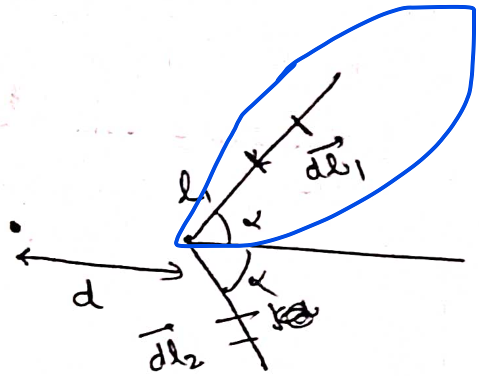
$$= \frac{\mu_0}{4\pi}\int_0^R \int_0^{2\pi} \frac{Kzr'\,(\cancel{-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}})\,dr'}{((r')^2+z^2)^{3/2}} + \frac{K(r')^2\, dr'\, d\phi'\,\hat{z}}{(r'^2+z^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\int_0^R \int_0^{2\pi} \frac{K(r')^2}{(r'^2+z^2)^{3/2}}\, dr'\, d\phi'\,\hat{z}$$

Substitute $r' = z\tan\theta$ $\{R = z\tan\alpha\}$

Final Ans $\rightarrow$ $\boxed{\frac{\mu_0 K}{2}\left[\ln(\sec\alpha + \tan\alpha) - \sin\alpha\right]}$

⑥



$$\vec{B} = \int d\vec{B_1} + \int d\vec{B_2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl_1} \times (\vec{r} - \vec{r_1}')}{|\vec{r} - \vec{r_1}'|^3} + \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl_2}' \times (\vec{r} - \vec{r_2}')}{|\vec{r} - \vec{r_2}'|^3}$$

$\vec{dl_2}' = dl_2' \cos\alpha\, \hat{n} - dl_2' \sin\alpha\, \hat{y}$ $\qquad \vec{dl_1} = -dl_1 \cos\alpha\, \hat{n} - dl_1' \sin\alpha\, \hat{y}$

$\vec{r} = -d\, \hat{n}$ $\qquad \vec{r} = -d\, \hat{n}$

$\vec{r_2}' = l_2' \cos\alpha\, \hat{n} - l_2' \sin\alpha\, \hat{y}$ $\qquad \vec{r_1}' = l_1' \cos\alpha\, \hat{x} + l_1' \sin\alpha\, \hat{y}$

$$\vec{B_1} = \frac{\mu_0 I}{4\pi}\int_0^\infty \frac{(-dl_1' \cos\alpha\, \hat{n} - dl_1' \sin\alpha\, \hat{y}) \times ((-d - l_1' \cos\alpha)\hat{n} - l_1' \sin\alpha)\hat{y}}{d^2 + (l_1')^2 + 2 d l_1' \cos\alpha}$$

$$= \frac{\mu_0 I}{4\pi}\int_0^\infty \frac{(l_1' \sin\alpha \cos\alpha - l_1' \sin\alpha \cos\alpha - d \sin\alpha)\, \hat{z} \cdot dl_1'}{(d^2 + (l_1')^2 + 2 d l_1' \cos\alpha)^{3/2}}$$

$$= -\frac{\mu_0 I d \sin\alpha\, \hat{z}}{4\pi} \int_0^\infty \frac{1}{[(l_1' + d\cos\alpha)^2 + (d\sin\alpha)^2]^{3/2}}\, dl_1'$$

$$\frac{l_1' \cancel{+ d\cos\alpha = \ldots}}{d\sin\alpha = \cancel{t}} \qquad \frac{l_1' + d\cos\alpha}{d\sin\alpha} = \tan\theta$$

$$dl_1' = \sec^2\theta\, d\theta\, (d\sin\alpha)$$

$$= \cancel{\frac{-\mu_0 I \hat{z}}{4\pi (d\sin\alpha)^2}}$$

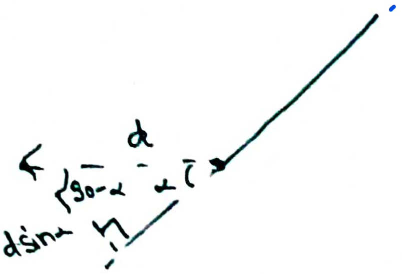
$$= \frac{-\mu_0 I\ \hat{z}}{4\pi (d\sin\alpha)^2} \int_{\frac{\pi}{2}-\alpha}^{\pi/2} \frac{\sec^2\theta \cdot d\theta}{\sec^3\theta} = \frac{-\mu_0 I (1-\cos\alpha)\, \hat{z}}{4\pi (d\sin\alpha)}$$

$$\vec{B_2} = \vec{B_1} = \frac{-\mu_0 I}{4\pi d} \frac{(1-\cos\alpha)\hat{z}}{\sin\alpha}$$

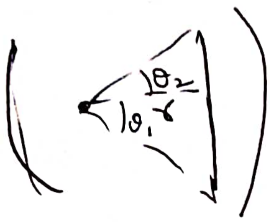
$$\boxed{\vec{B} = \frac{-\mu_0 I}{2\pi d} \tan\frac{\alpha}{2}\, \hat{z}}$$

Additional remarks about Q6

(i) JEE DAYS !!



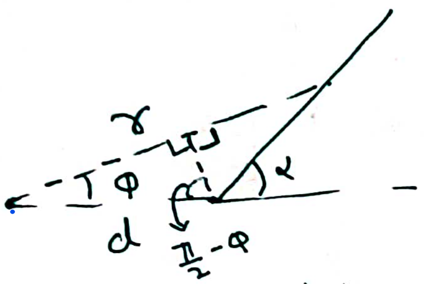
$$B = \frac{\mu_0}{4\pi (d\sin\alpha)}(\sin\theta_1 + \sin\theta_2) = \frac{\mu_0}{4\pi}\frac{\left(\sin(-(90-\alpha)) + \sin 90^\circ\right)}{d\sin\alpha}$$



$$= \frac{\mu_0}{4\pi}\frac{(1-\cos\alpha)}{d\sin\alpha}$$

(ii) Could also be solved using result of 2(a)

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{d\phi}{r(\phi)} = \frac{\mu_0 I}{4\pi}\int_0^{\alpha}\frac{d\phi}{r(\phi)}$$

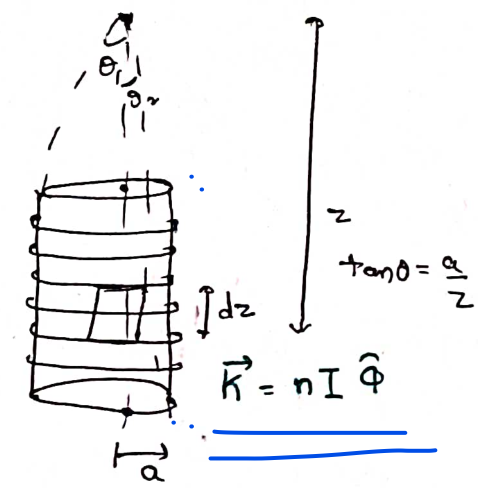


$$r = d\cos\phi + \frac{d\sin\alpha}{\cot \cancel{\tan}\left(\pi - \alpha - \frac{\pi}{2} + \phi\right)} = d\cos\phi + \frac{d\sin\alpha \,\cos\cancel{\sin}(\alpha-\phi)}{\sin\cancel{\cos}(\alpha-\phi)}$$

$$= \frac{d\sin\alpha}{\sin(\alpha-\phi)}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{\alpha}\frac{d\phi\,\sin(\alpha-\phi)}{d\sin\alpha} = \frac{\mu_0 I}{4\pi d\sin\alpha}(1-\cos\alpha)$$

(9)



$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,da$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{(nI\hat{\phi})\times(z\hat{z} - a\hat{r} - \phi\hat{\phi})}{(a^2+z^2)^{3/2}}\,a\,d\phi\,dz$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{(nIz\,\hat{r} - nIa\hat{z})}{(a^2+z^2)^{3/2}}\,a\,d\phi\,dz$$

$$\tan\theta = a/z$$

$$\sec^2\theta\,.d\theta = -\frac{a}{z^2}dz = -\tan^2\theta\,\frac{dz}{a}$$

$$dz = \frac{-a}{\sin^2\theta}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\int_0^{2\pi}\int_{\theta_1}^{\theta_2} \frac{nIa}{\tan\theta}\,\frac{(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})}{a^2\,\mathrm{cosec}^3\theta}\,d\phi\left(\frac{-a^2}{\sin^2\theta}\right)d\theta \right.$$

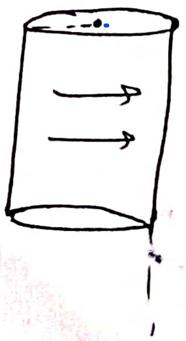
$$\left. + \int_0^{2\pi}\int_{\theta_2}^{\theta_1} \frac{nIa\,\hat{z}}{a^3\,\mathrm{cosec}^3\theta}\left(\frac{a^2}{\sin^2\theta}\right).\,d\phi\,d\theta\right]$$

$$\vec{B} = \frac{\mu_0 nI}{4\pi}\int_0^{2\pi} d\phi\,.\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta\,\hat{z} = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)\,\hat{z}$$

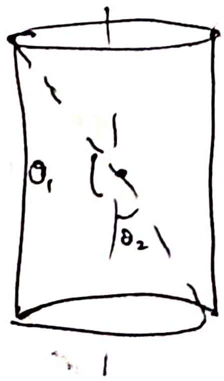
(i) Half solenoid

$\theta_1 = \pi/2$, $\theta_2 = 0$

$$\vec{B} = \frac{\mu_0 nI}{2}\hat{z}$$



(ii) Infinite solenoid



$\theta_1 \to \pi$

$\theta_2 \to 0$

$\vec{B} = \mu_0 n i\,\hat{z}$