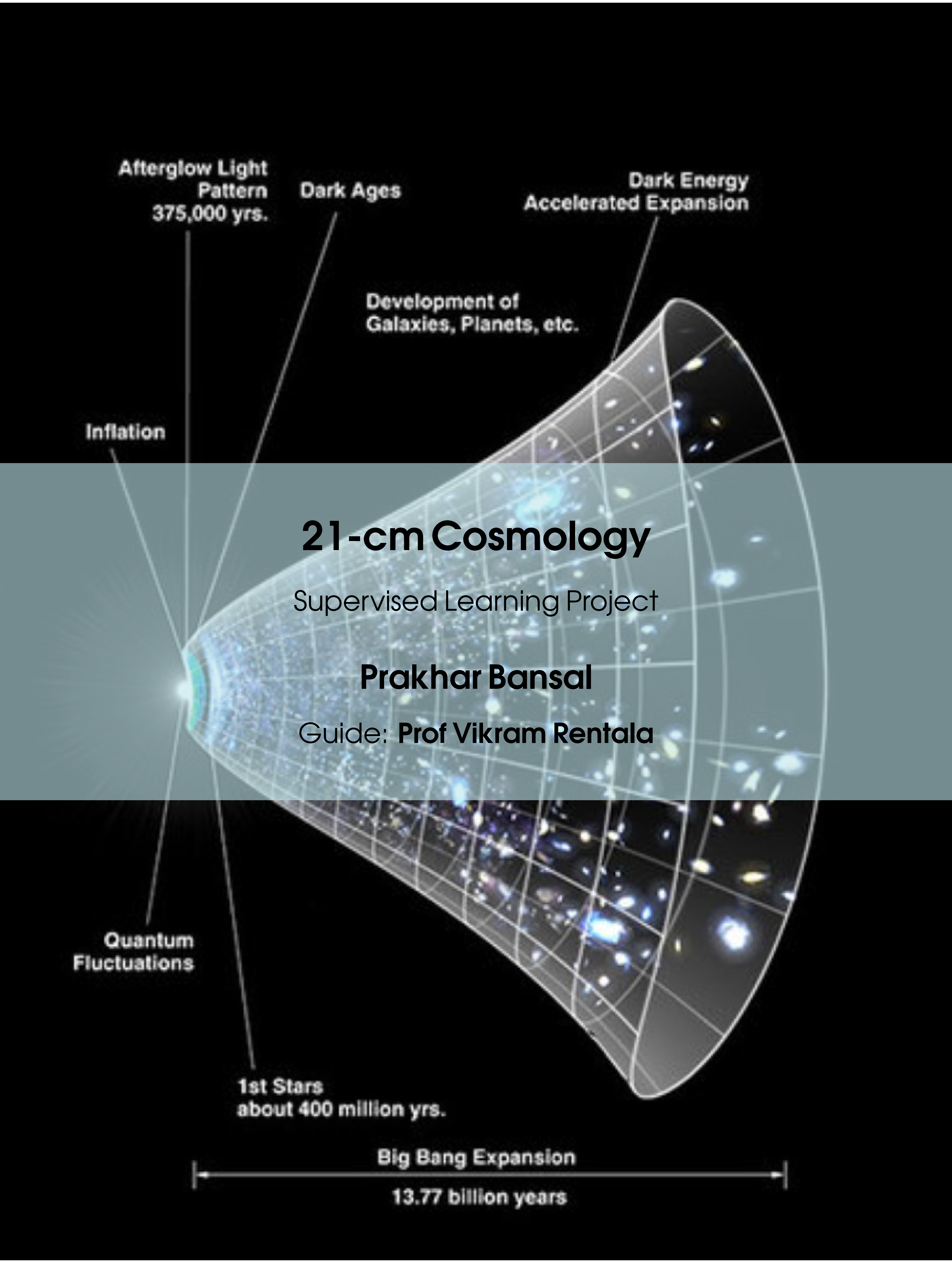
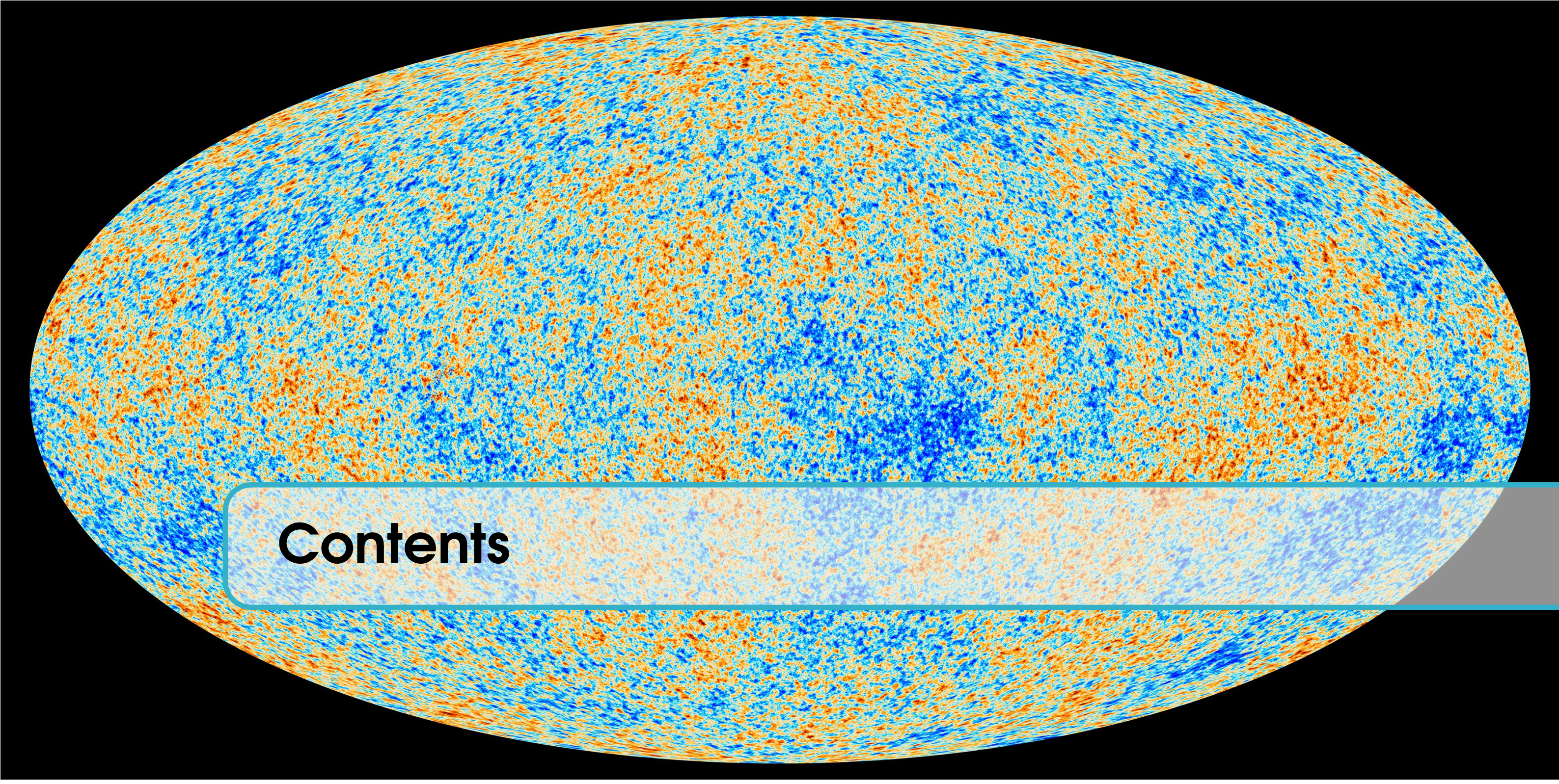


21-cm Cosmology

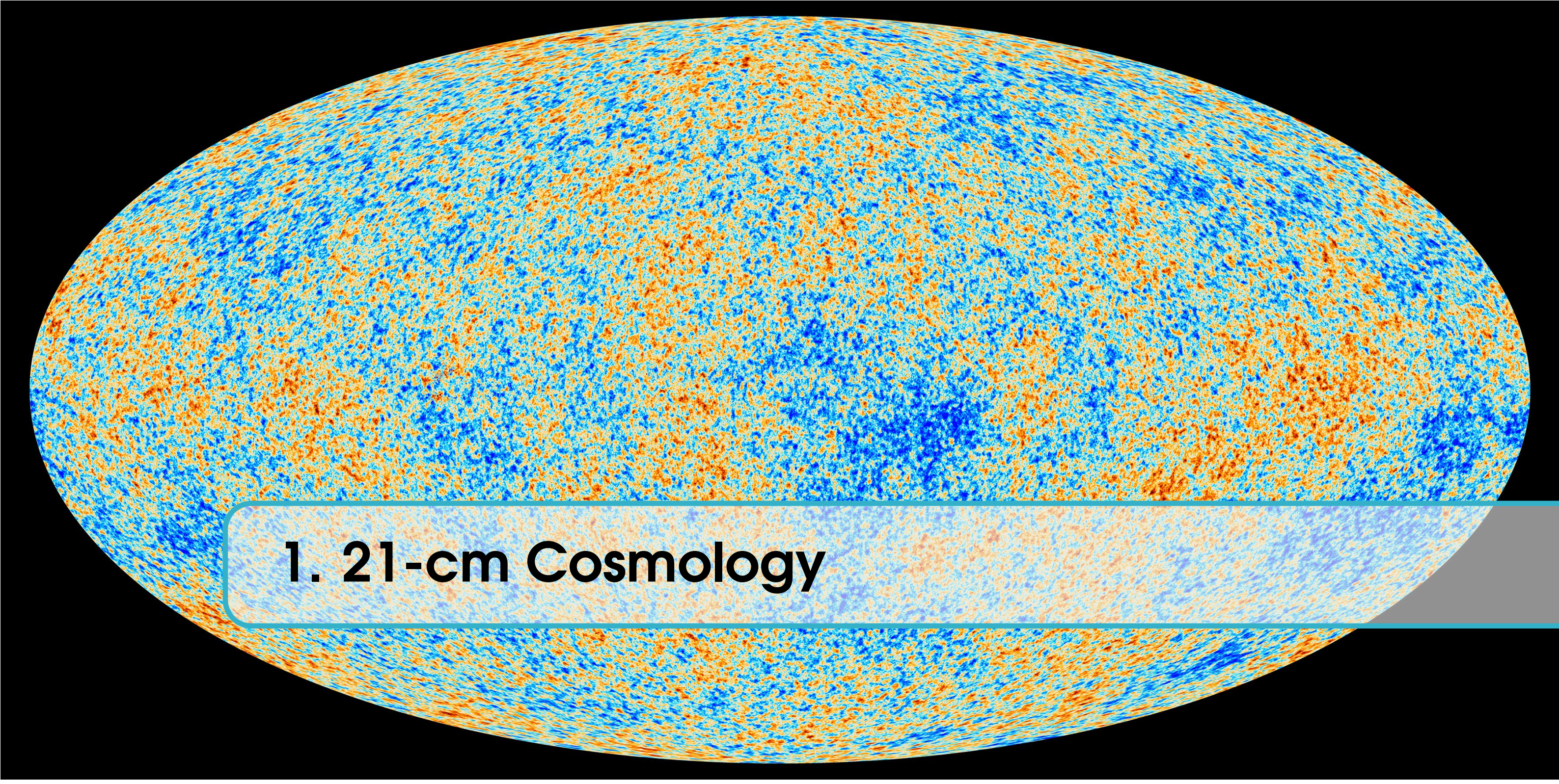
Supervised Learning Project

Prakhar Bansal

Guide: **Prof Vikram Rentala**



Contents



1. 21-cm Cosmology

1.1 Introduction

With the advent of several ground based and space based telescopes, we have been able to directly observe the galaxies out to distances corresponding to a time when the Universe was a billion years old. The detection of CMB in 1964 allowed us to probe the primordial universe when it was just 40,000 years old. However when it comes to the observation of the era connecting the two aforementioned periods, the picture still remains blurry. This middle era is possibly one of the most interesting epochs in the evolution of the universe when the first stars and galaxies came into existence.

In this report I have studied the basics of the field of 21-cm Cosmology, which is an alternative attempt to indirectly observe the first stars and galaxies using the 21-cm signal emitted by the abundant neutral-H in the early universe. The signal is usually studied as an imprint to the background radio sources like CMB or Quasars. With the development of several recent attempts like James Webb Space Telescope(JWST) to observe the universe at very high redshifts, the field of 21-cm Cosmology becomes an exciting area where we'll get to test our theoretical predictions against the observations and study the birth of first stars and galaxies.

1.2 Radiative Transfer Equations

1.2.1 Bightness Temperature

Planck's law gives us the specific intensity distribution of a blackbody in terms of the frequency (or wavelength) of the radiation emitted by the body

$$I_s(\nu) = \frac{2h\nu^3}{c^2} . \frac{1}{e^{\frac{h\nu}{k_b T}} - 1} \tag{1.1}$$

In the limit $h\nu << k_b T$ the above relation reduces to

$$I_s(\nu) \approx \frac{2h\nu^3}{c^2} . \frac{1}{1 + \frac{h\nu}{k_b T} - 1} \tag{1.2}$$

$$= \frac{2k_b T \nu^2}{c^2} \tag{1.3}$$

This limit is known as Rayleigh Jeans limit of small frequencies.

This temperature T is used as a proxy for the specific intensity and is termed as the **Brightness Temperature** of the radiation

1.2.2 Spin Temperature

The opposite and same spins of the proton and the electron in a neutral H atom interact and give rise to the singlet and triplet states respectively as shown

$$00 = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \tag{1.4}$$

$$1,-1 = \downarrow\downarrow, 1,0 = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), 1,1 = \uparrow\uparrow \tag{1.5}$$

The energy difference between the two state is $\Delta = 5.9\mu eV$ corresponding to a photon of 21 cm wavelength. In a sample of neutral H at equilibrium, the ratio of number of atoms in triplet to singlet state is given by

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{\Delta}{k_B T_S}} = 3e^{-\frac{\Delta}{k_B T_S}} \tag{1.6}$$

where T_S is a characteristic temprature of the sample known as **Spin Temperature**

1.2.3 Diving into the equations

In cosmological contexts the 21 cm line has been used as a probe of gas along the line of sight to some background radio source. The basic equation of radiative transfer for the specific intensity I_ν ($\frac{dI}{d\nu d\Omega}$)in the absence of scattering along a path described by coordinate s

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \tag{1.7}$$

where absorption and emission by gas along the path are described by the coefficients α_ν and j_ν, respectively. Using the standard definition of optical depth $d\tau_\nu = \alpha ds$ and writing $j_\nu/\alpha = S_\nu$ we obtain

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \tag{1.8}$$

S_ν is the intensity emitted by the source

We mainly consider **Background Radiation** from the following two sources

- **CMB** - In this case, $I_\nu = B_\nu(T_{CMB})$ and the 21 cm feature is seen as a spectral distortion to the CMB blackbody at appropriate radio frequencies
- **Radio Loud Point Source** - Eg Quasar

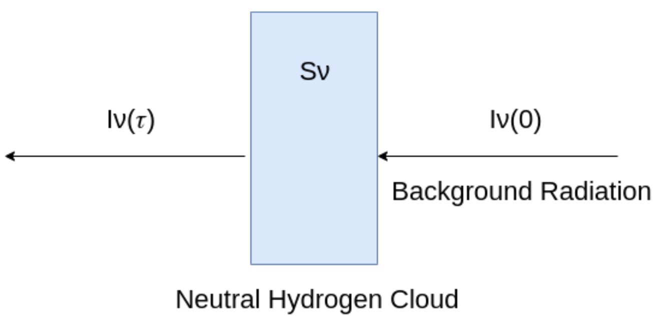


Figure 1.1: Process of Radiative Transfer

Source Term- The source term that we consider is the neutral H cloud through which the background radiation passes. Since CMB have energies in radio regime they can interact with neutral H only through the hyperfine transition, hence the radiation from the source which is of primary interest to us is the one having characteristic brightness temperature as spin temperature

$$S_\nu = B_\nu(T_S) = \frac{2k_b\nu^2 T_S}{c^2} \tag{1.9}$$

To simplify the expressions, we work in Rayleigh Jeans limit. Substituting (3) in (5) and cancelling $\frac{2k_b\nu^2}{c^2}$ from both sides we get

$$\frac{dT_B}{d\tau_\nu} = -T_B + T_S$$
$$\frac{dT_B}{d\tau_\nu}e^{\tau_\nu} + T_B e^{\tau_\nu} = T_S e^{\tau_\nu}$$
$$\int_0^{\tau_\nu}\left(\frac{dT_B}{d\tau_\nu}e^{\tau_\nu} + T_B e^{\tau_\nu}\right)d\tau_\nu = \int_0^{\tau_\nu} T_S e^{\tau_\nu} d\tau_\nu$$
$$(T_B e^{\tau_\nu})\Big|_0^{\tau_\nu} = T_S(e^{\tau_\nu})\Big|_0^{\tau_\nu}$$
$$T_B(\tau_\nu)e^{\tau_\nu} - T_B(0) = T_S(e^{\tau_\nu} - 1)$$
$$\boxed{T_B(\tau_\nu) = T_B(0)e^{-\tau_\nu} + T_S(1 - e^{-\tau_\nu})} \tag{1.10}$$

where
T_B- Brightness temperature of background radio source
T_S- Spin Temperature of Neutral H Cloud

In case of an optically thin cloud ($\tau_\nu << 1$)

$$T_B(\tau_\nu) - T_B(0) = -T_B(0)\tau_\nu + T_S\tau_\nu \tag{1.11}$$
(1.12)

If the cloud is present at a redshift z, the temperatures get redshifted by (1+z)

$$\boxed{\delta T_B = \frac{T_S - T_B}{1+z}\tau_\nu} \tag{1.13}$$

1.3 Spin Temperature of H-atom

The key to the detectability of the 21 cm signal hinges on the spin temperature.
As can be seen from the expression for Brightness Temperature, only if the spin temperature deviates from the background temperature, will a signal be observable. In the following sections we first understand the microscopic phenomena which decides the spin temperature of a neutral H sample and then establish the relation with the macroscopic parameters like absorption and emission coefficients that appear in Radiative Transfer Equations

1.3.1 Factors affecting Spin Temperature

Three processes determine the spin temperature:

1. Absorption/emission of 21 cm photons from/to the radio background, primarily the CMB;
2. Collisions with other hydrogen atoms and with electrons;
3. Resonant scattering of Lyα photons that cause a spin flip via an intermediate excited state

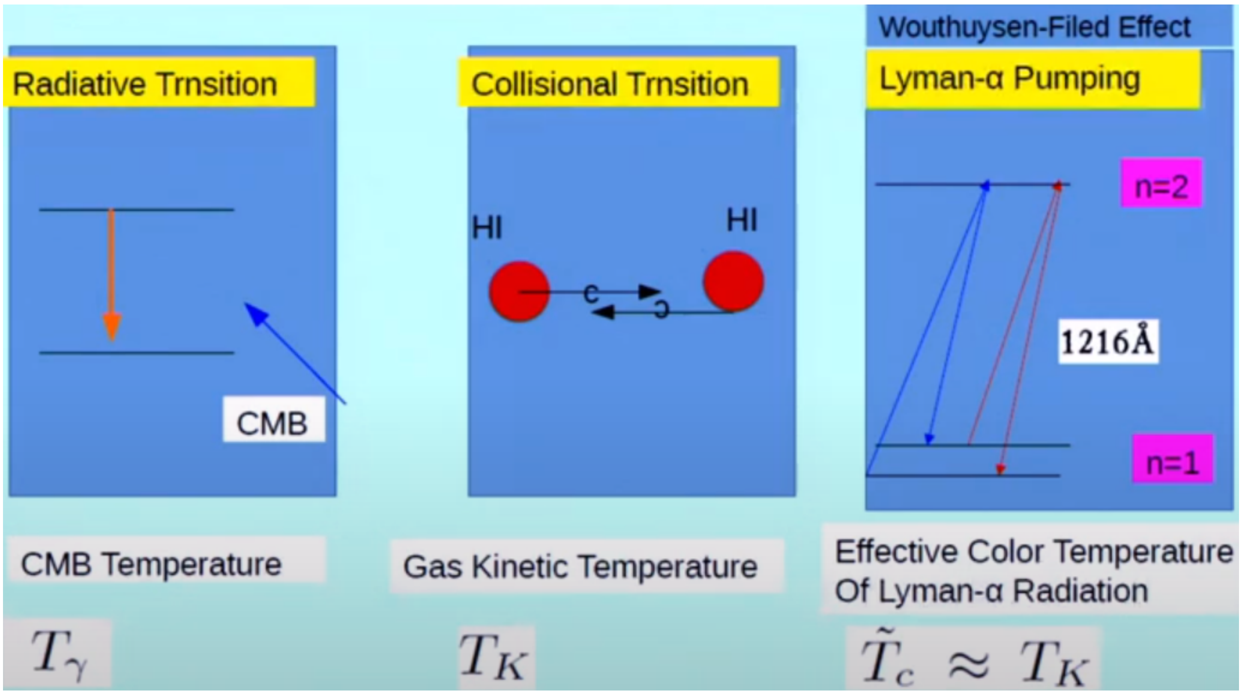


Figure 1.2: Factors affecting Spin Temperature

Each of the above process tries to bring the spin temperature equal to its characteristic temperature $T_\gamma, T_K, \tilde{T}_C$

1.3.2 Rate Equation

$$\dot{n}_0 = -(P^\gamma_{01} + P^C_{01} + P^\alpha_{01})n_0 + (P^\gamma_{10} + P^C_{10} + P^\alpha_{10})n_1 \tag{1.14}$$

Detailed Balance

At equilibrium, each elementary process is in equilibrium with its reverse process. Consider a situation where radiative tranistion (due to CMB) dominate ($T_S = T_\gamma$). At equilibrium

$$\dot{n}_0 = 0$$
$$n_0 P^\gamma_{01} = n_1 P^\gamma_{10}$$
$$\frac{P^\gamma_{01}}{P^\gamma_{10}} = \frac{n_1}{n_0} = 3e^{-\frac{T_*}{T_\gamma}}$$
$$\boxed{\frac{P^\gamma_{01}}{P^\gamma_{10}} \approx 3(1 - \frac{T_*}{T_\gamma})} \tag{1.15}$$

where $T_* = \Delta/k_B$

We can obtain similar relations for rate constants corresponding to Collisional Coupling and Lyα Coupling

$$\boxed{\frac{P^C_{01}}{P^C_{10}} \approx 3(1 - \frac{T_*}{T_K})} \tag{1.16}$$

$$\boxed{\frac{P^\alpha_{01}}{P^\alpha_{10}} \approx 3(1 - \frac{T_*}{T_C})} \tag{1.17}$$

Applying equilibrium condition ($\dot{n}_0 = 0$) in (1.14) , we obtain

$$\frac{n_1}{n_0} = \frac{P_{01}^{\gamma} + P_{01}^{C} + P_{01}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}}$$

$$3(1 - \frac{T_*}{T_S}) = \frac{3(1 - \frac{T_*}{T_\gamma})P_{10}^{\gamma} + 3(1 - \frac{T_*}{T_K})P_{10}^{C} + 3(1 - \frac{T_*}{T_C})P_{10}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}} \tag{1.18}$$

$$T_S^{-1} = \frac{T_\gamma^{-1} P_{10}^{\gamma} + T_K{}^{-1} P_{10}^{C} + T_C{}^{-1} P_{10}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}}$$

$$\boxed{T_S^{-1} = \frac{T_\gamma^{-1} + T_K^{-1} x_C + T_C^{-1} x_\alpha}{1 + x_C + x_\alpha}} \tag{1.19}$$

$$x_C = \frac{P_{10}^{C}}{P_{10}^{\gamma}}$$

$$x_\alpha = \frac{P_{10}^{\alpha}}{P_{10}^{\gamma}}$$

1.3.3 Coupling Coefficients

1.3.3.1 Collisional Coupling

Three main channels are available for collisional coupling -:

1. collisions between two hydrogen atoms
2. collisions between a hydrogen atom and an electron or
3. a proton

$$x_C = x_C^{HH} + x_C^{eH} + x_C^{pH}$$

$$\boxed{x_C = \frac{T_*}{A_{10} T_\gamma} \left(\kappa_{1-0}^{HH}(T_k) n_H + \kappa_{1-0}^{eH}(T_k) n_e + \kappa_{1-0}^{pH}(T_k) n_p \right)} \tag{1.20}$$

where κ_{1-0}^{Hi} is the scattering rate for the collision with i th species and A_{10} is the Einstein coefficient for Spontaneous Emission

1.3.3.2 Lyman-α Coupling

$$\boxed{x_\alpha = \frac{S_\alpha J_\alpha}{1.165 \times 10^{10} [(1+z)/20] cm^{-2} s^{-1} Hz^{-1} sr^{-1}}} \tag{1.21}$$

where S_α is a factor of order unity and J_α is specific flux ($dN/dAdtd\nu d\Omega$) of Lymanα photons

1.3.4 Line Profile

Owing to Heisenberg's uncertainity relation the energy difference between the two levels is not infinitely sharp but is described by a line profile function $\phi(\nu)$ which is sharply peaked at $\nu = \nu_o$ and which is conveniently taken to be normalized. This line profile function describes the relative effectiveness of frequencies in the neighborhood of ν_o for causing transitions.

$$\int \phi(\nu) d\nu = 1$$

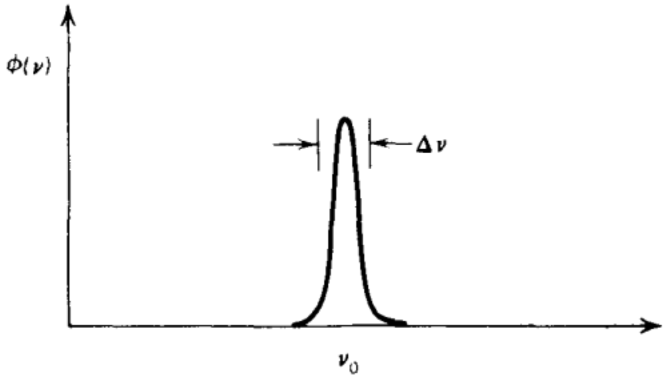


Figure 1.3: Line Profile Function

1.3.5 Einstein's Coefficients

The relationship between macroscopic absorption and emission coefficients α_ν and j_ν (1.7) and absorption and emission at microscopic level was first studied by Einstein. Three processes were identified

1. Spontaneous Emission
2. Absorption
3. Stimulated Emission

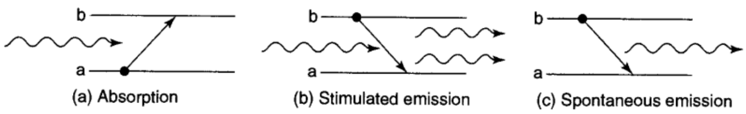


Figure 1.4: Three types of Radiative Transition

- **Spontaneous Emission Coefficient**
  $A_{10} =$ transition probability per unit time for spontaneous emission(s^{-1})
- **Absorption Coefficient**
  Since the transition occours in the presence of photons of energy $h\nu_o$(and nearby), we expect that the probability per unit time for this process will be proportional to the density of photons (or to the mean intensity) at frequency ν_o
  We express the transition probability per unit time as $B_{01}\bar{J}$
  where

$$\bar{J} = \int J_\nu \phi(\nu) d\nu \qquad J_\nu = \int \frac{I_\nu}{4\pi} d\Omega \tag{1.22}$$

  For an isotropic radiation $J_\nu = I_\nu$
  In case of most of the radiations J_ν changes slowly over the width $\Delta\nu$ of the line, $\phi(\nu)$ behaves like a δ function thus for an isotropic such radiation rate of transition probabilitiy is simply $B_{01}I_\nu$

- **Stimulated Emission Coefficient**
  Rate of Transition Probability $=B_{10}I_\nu$

1.3.5.1 Relation between Einstein Coefficients

In thermal equilibrium

$$n_1 A_{10} + n_1 B_{10} I_\nu = n_0 B_{01} I_\nu$$
$$I_\nu = \frac{A_{10}/B_{10}}{(n_0 B_{01})/(n_1 B_{10}) - 1}$$
$$I_\nu = \frac{A_{10}/B_{10}}{(g_0 B_{01}/g_1 B_{10}) e^{\frac{\Delta}{k_B T_S}} - 1}$$

n_1, n_0 is the number density of atoms in respective energy levels
Comparing with 1.1 we get

$$A_{10} = \frac{2h\nu^3}{c^2} B_{10} \tag{1.23}$$
$$g_0 B_{01} = g_1 B_{10} \tag{1.24}$$

1.3.6 Connection with Macroscopic Coefficients

- **Emission Coefficient** j_ν
  The amount of energy emitted in volume dV, solid angle $d\Omega$, frequency range $d\nu$, and time dt is, by definition, $j_\nu dV d\Omega d\nu dt$

  Each atom contributes an energy $h\nu_o$, distributed over 4π solid angle for each transition. Thus total energy emitted may also be expressed as $(h\nu_o/4\pi)\phi(\nu) n_1 A_{10} dV d\Omega d\nu dt$. This implies

$$\boxed{j_\nu = (h\nu_o/4\pi)\phi(\nu) n_1 A_{10}} \tag{1.25}$$

- **Absorption Coefficient** α_ν
  Using similar arguments and from the second term of 1.7 we obtain

$$\alpha_\nu I_\nu dV d\Omega d\nu dt = (h\nu_o/4\pi)\phi(\nu)(n_0 B_{01} - n_1 B_{10}) I_\nu dV d\Omega d\nu dt$$
$$\boxed{\alpha_\nu = (h\nu_o/4\pi)\phi(\nu)(n_0 B_{01} - n_1 B_{10})} \alpha_\nu = (h\nu_o/4\pi)\phi(\nu) n_0 B_{10}(B_{01}/B_{10} - n_1/n_0)$$
$$\alpha_\nu = \frac{h\nu_o}{4\pi}\phi(\nu) n_0 \frac{c^2}{2h\nu_o^3} A_{10}\left(3 - 3e^{\frac{-T_*}{T_S}}\right)$$
$$\alpha_\nu \approx \frac{h\nu_o}{4\pi}\phi(\nu) n_0 \frac{c^2}{2h\nu_o^3} A_{10}\left(3\frac{T_*}{T_S}\right) \tag{1.26}$$

Since $n_1 = n_0 e^{\frac{-T_*}{T_S}}$ and $T_* = 0.068K << T_S \sim T_\gamma = 2.73K$

$$n_o \approx n_{HI}/4$$

$$\implies \boxed{\alpha_\nu \approx \frac{3c^2 A_{10}}{32\pi\nu_0^2} n_{HI}\left(\frac{T_*}{T_S}\right)\phi(\nu)} \tag{1.27}$$

1.4 Optical Depth

We use the expression of absorption coefficient to calculate the optical depth of a neutral-H cloud of length l

$$\tau_\nu = \int \alpha_\nu dl$$
$$\tau_\nu = \int \frac{3c^2A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \phi(\nu) dl$$
$$\tau_\nu = \frac{3c^2A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \int_{\text{length of cloud}} \phi(\nu) dl$$
$$\text{Using} \int \phi(\nu) d\nu = 1$$
$$\boxed{\phi(\nu) \approx \frac{1}{\Delta\nu}}$$
$$\tau_\nu = \frac{3c^2A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \frac{\Delta l}{\Delta\nu} \tag{1.28}$$

Consider a neutral-H cloud moving away from observer with velocity v as shown below. Due to this velocity the radiation emitted by the cloud at $\nu_o = 1420MHz$ is redshifted to ν given by the formula

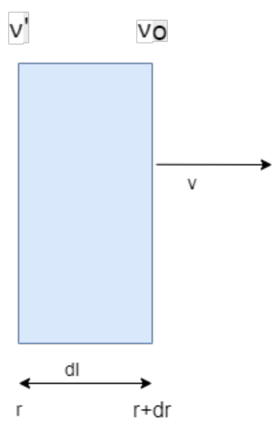


Figure 1.5: A moving neutral-H cloud

$$\nu' = \nu_o(1 - v/c)$$
$$\Delta\nu = \nu_o(v/c)$$
$$\Delta l = a\Delta r$$
$$v = \frac{\Delta l}{\Delta t} = \dot{a}\Delta r + \frac{\partial v}{\partial r}\Delta r$$
$$\frac{\Delta l}{\Delta \nu} = \frac{a\Delta r}{\nu_o v/c} = \frac{c}{\nu_o}\frac{a\Delta r}{\dot{a}\Delta r + \frac{\partial v}{\partial r}\Delta r}$$
$$= \frac{c}{\nu_o H}\left(1 + \frac{1}{aH}\frac{\partial v}{\partial r}\right)^{-1}$$
$$\boxed{\frac{\Delta l}{\Delta \nu} \approx \frac{c}{\nu_o H}\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right)} \quad (1.29)$$
$$\tau = \frac{3c^3 A_{10}}{32\pi\nu_0^3 H} n_{HI}\left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \quad (1.30)$$
$$\tau = (1+z)\hat{T}(z)\frac{\rho_{HI}}{\rho_H}\left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \quad (1.31)$$

where we combine all prefactors in $\hat{T}(z)$

$$\hat{T}(z) = 4mK(1+z)^2\frac{\Omega_b h^2}{0.02}\frac{0.7}{h}\frac{H_o}{H_a} \quad (1.32)$$

From 1.13 the differential brightness temperature is

$$\delta T_B = \left(1 - \frac{T_B}{T_S}\right)\hat{T}(z)\frac{\rho_{HI}}{\rho_H}\left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \quad (1.33)$$

1.5 Global 21-cm signal

The global 21cm signal is the angle averaged version of the the 21cm signal coming from all directions of the sky. Due to this angle averaging the $\frac{\partial v}{\partial r}$ in 1.33 becomes zero and we obtain

$$\delta T_B = \left(1 - \frac{T_B}{T_S}\right)\hat{T}(z)\frac{\rho_{HI}}{\rho_H}\left(\frac{T_*}{T_S}\right) \quad (1.34)$$

Three cases arise

1. $T_S > T_B \implies \delta T_B > 0$, Emission Signal
2. $T_S < T_B \implies \delta T_B < 0$, Absorption Signal
3. $T_S = T_B \implies \delta T_B = 0$, No Signal

1.5.1 Evolution of the Global 21-cm Signal

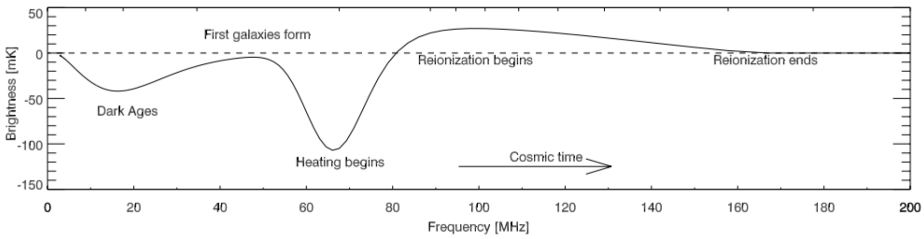


Figure 1.6: Expected evolution of the sky-averaged 21-cm brightness from the "dark ages" at redshift 200 to the end of reionization, sometime before redshift 6. Fig Credits: "21-cm cosmology in the 21st Century" Pritchard and Loeb, 2012

The brightness temperature can be expressed as $T_B = T_B(T_K, x_i, J_\alpha, n_H)$. As our universe evolves, it passes through through certain epochs where the values of these quantities change quite drastically. This leads to a significant (measurable) variation in Brightness Temperature fluctuations across various redshifts as shown in 1.6. Broadly the various phases in the evolution of the global 21-cm signal are classified as follows

- $\mathbf{200 \lesssim z \lesssim 1100}$**:** The residual free electron fraction left after recombination allows Compton scattering to maintain thermal coupling of the gas to the CMB, setting $T_K = T_\gamma$. The high gas density leads to effective collisional coupling so that $T_S = T_\gamma$ and we expect $\delta T_B = 0$ and no detectable 21 cm signal.
- $\mathbf{40 \lesssim z \lesssim 200}$**:** In this regime, the gas cools adiabatically so that $T_K \propto (1+z)^2$ leading to $T_K < T_\gamma$ and collisional coupling sets $T_S < T_\gamma$, leading to $\delta T_B < 0$ and an early absorption signal.
- $\mathbf{z_* \lesssim z \lesssim 40}$: Due to expansion the gas density decreases and collisional coupling becomes ineffective. The spin temperature starts coupling to the CMB again resulting in $T_s = T_\gamma$ and hence no detectable signal.
- $\mathbf{z_\alpha \lesssim z \lesssim z_*}$: Once the first sources switch on at z_*, they emit both $Ly-\alpha$ photons and X-rays. In general, the emissivity required for $Ly-\alpha$ coupling is significantly less than that for heating T_K above T_γ. We therefore expect a regime where the spin temperature is coupled to cold gas so that $T_S \sim T_K < T_\gamma$ and there is an absorption signal.
- $\mathbf{z_h \lesssim z \lesssim z_\alpha}$: As the heating starts T_K starts to increase and becomes equal to T_γ by the redshift z_h. All throughout this epoch δT_B rises and becomes eventually equal to 0
- $\mathbf{z_T \lesssim z \lesssim z_h}$: After the heating transition, $T_K > T_\gamma$ and we expect to see a 21 cm signal in emission
- $\mathbf{z_r \lesssim z \lesssim z_T}$: Continued heating drives $T_K >> T_\gamma$ at z_T and δT_B saturates.
- $\mathbf{z \lesssim z_r}$: Once reionization begins (zz_r) the neutral H fraction $\frac{\rho_{HI}}{\rho_H}$ starts to decrease rapidly and the brightness temperature fluctuations starts approaching zero.

Note that there is considerable uncertainty in the exact form of this signal, arising from the unknown properties of the first galaxies. The prediction of the properties of this signal is an active area of research in the field of 21-cm Cosmology. In the next phase of the project, under Prof Rentala's guidance I will study the role of dark matter in sourcing the fluctuations in the 21-cm Signal.

1.6 Summary

Neutral-H clouds leave an imprint on background radio signals (like CMB or Quasars) in the form of temperature fluctuations. These temperature fluctuations depend on the spin temperature of the cloud. The spin temperature couples to the brightness temperature of background radio source or Kinetic Temperature of H gas depending on the neutral H fraction. As our universe evolves, it passes through through certain epochs where the values of the spin temperature change quite drastically. This leads to a significant (measurable) variation in Brightness Temperature fluctuations across various redshifts

1.7 References

- "21-cm cosmology in the 21st Century" Pritchard and Loeb, 2012
- "Radiative Processes in Astrophysics" Rybicki and Lightman, 1985
- 21cm Physics and Cosmology Lectures by Somnath Bharadwaj at ICTS