

Parameter Constraints

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1 Introduction

I have reproduced the plots given in the original paper Cao et al (2203.10825) for Non-Flat *LCDM* and *XCDM* models. For each model I used emcee to compute the Markov chains and use them to calculate 1D marginalized distributions corresponding to each free parameter. Here I first present my results for LCDM Model and then XCDM Model.

2 Non-Flat LCDM Model

2.1 Data

Since the original paper Cao et al (2203.10825) contains the combined results for H(z)+BAO data given in table 1 and 2 of the paper so I have also done the analysis for H(z)+BAO dataset so as to compare my results.

2.2 Method

First I wrote the code for χ^2 function and then minimized it using `scipy.minimize`. It returns the minimized value of function χ_{min}^2 and the best fit value of free parameters $\Omega_{k0}, \Omega_b h^2, \Omega_c h^2, H_o$ which are the same as used in the paper. From these free parameters we can also calculate the best fit value of derived parameter $\Omega_m = \frac{\Omega_b h^2 + \Omega_c h^2 + \Omega_\nu h^2}{h^2}$ using the value of neutrino density parameter, $\Omega_\nu h^2 = 0.06/93.14$ given in the paper. Then I ran the emcee code to obtain Markov chains for these free parameters and the derive parameter Ω_m . While running the mcmc simulation for all the parameters (except H_o) I used the same priors as in the paper. For H_o the paper uses the priors from 0 to ∞ while I used the priors (as suggested in Ryan's thesis) from 50 to 85. These set of priors were used to save time while running the MCMC simulations. I ran the simulations for 5000 steps and for 200 walkers.

2.3 Comparison of Results

In the following table, I have made a comparison of results of my work with that of original paper.

Note: The paper contains only parameter values and χ_{min}^2 for the H(z)+BAO analysis but not the plots. So I've included the plots which contain analysis of H(z)+BAO+SN data (in red coloured contours in figure 1).

Attribute	Cao et al	Using emcee and scipy
$\Omega_b h^2$	0.026	0.026
$\Omega_c h^2$	0.1098	0.1097
Ω_m	0.292	0.2917
Ω_k	0.048	0.0477
H_o	68.35	68.37
χ_{min}^2	25.30	25.28
AIC	33.30	33.28
BIC	40.43	40.42

Table 1: Unmarginalized Best Fitting Parameter Values for BAO+H(z) data

Attribute	Cao et al	Using emcee and scipy
Plots	Figure 1	Figure 2
$\Omega_b h^2$	$0.0266^{+0.0039}_{-0.0045}$	$0.0275^{+0.0035}_{-0.0057}$
$\Omega_c h^2$	0.1088 ± 0.0166	0.107 ± 0.017
Ω_m	0.291 ± 0.023	0.289 ± 0.022
Ω_k	$0.059^{+0.081}_{-0.091}$	$0.064^{+0.079}_{-0.090}$
H_o	68.37 ± 2.10	68.4 ± 2.0

Table 2: One-dimensional marginalized posterior mean values and uncertainties ($\pm 1\sigma$ error bars or 2σ limits) of the parameters

2.4 Plots

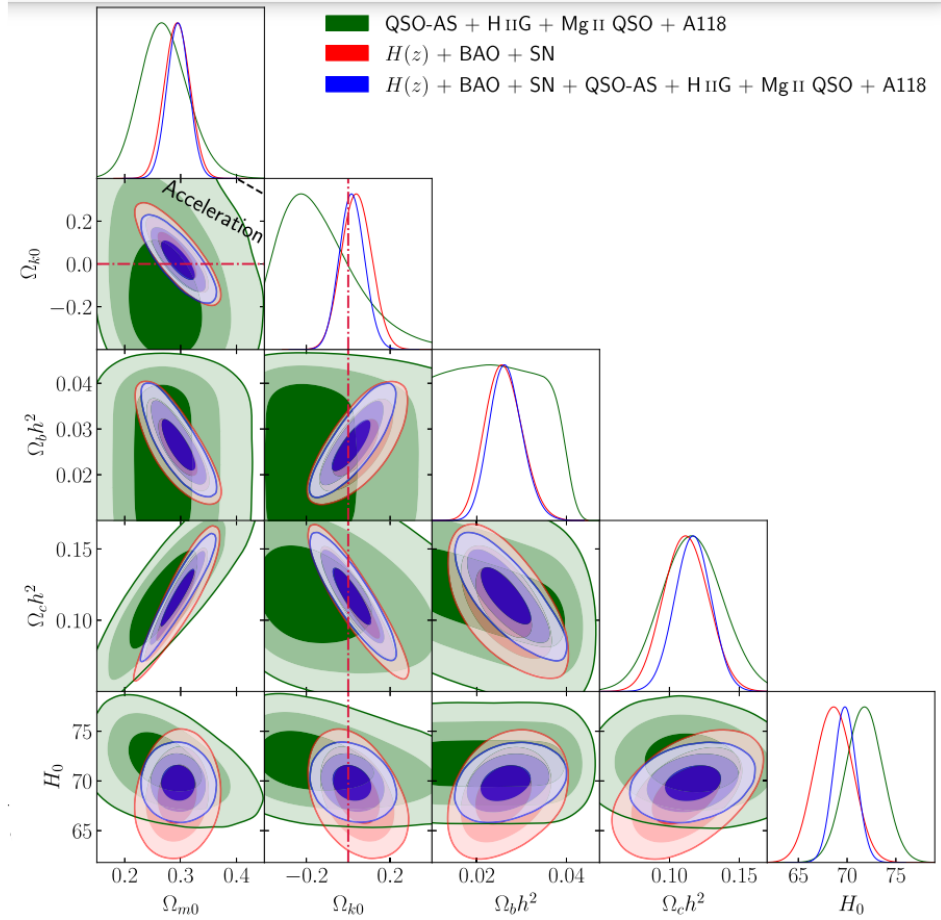


Figure 1: Original Plots in Cao et al

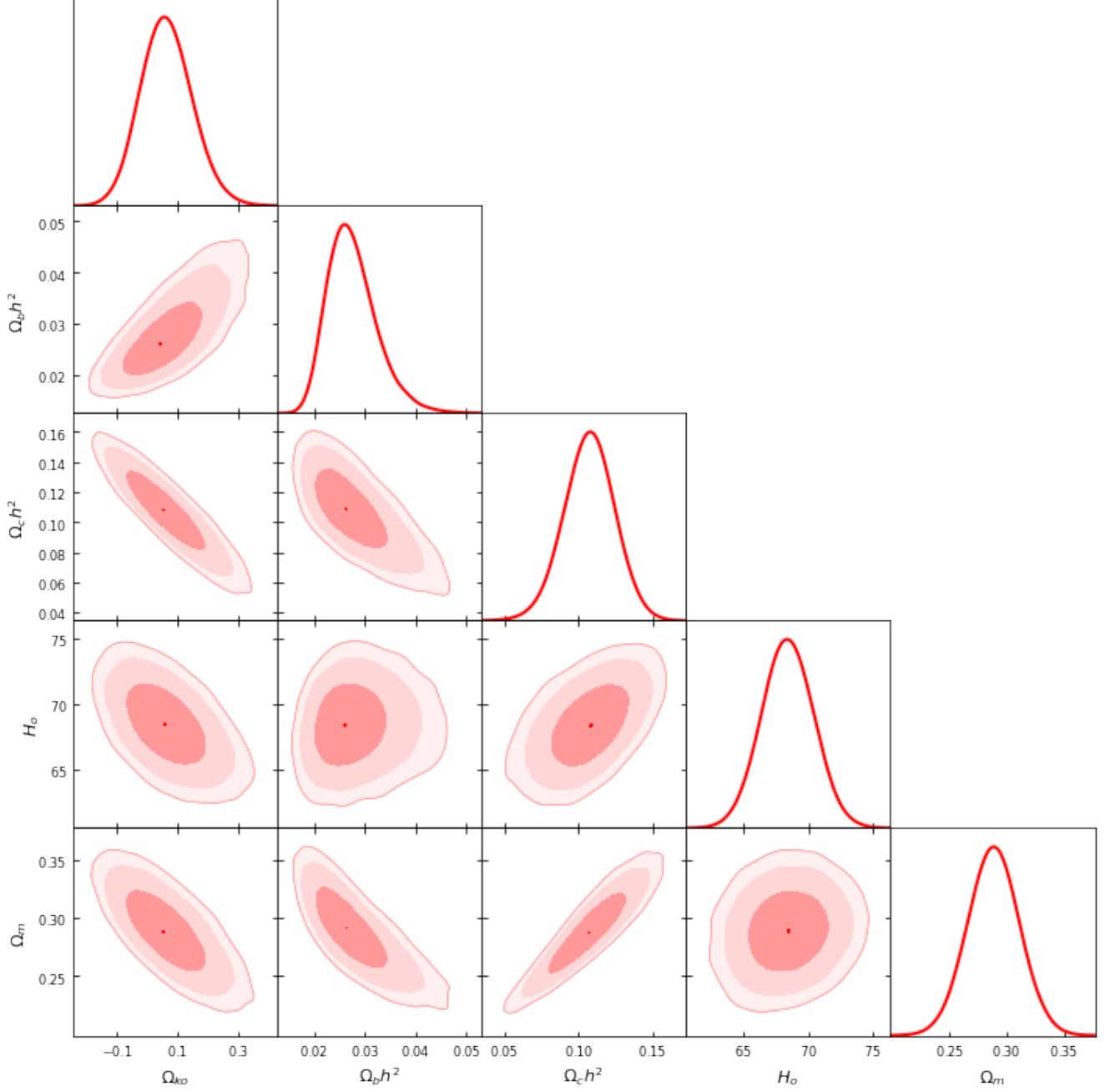


Figure 2: Plots generated using emcee and scipy

3 Non-Flat XCDM Model

3.1 Data

Same as used above

3.2 Method

The whole methodology remains same as used for Non-Flat LCDM Model. Except here the number of free parameters have increased from four to five ($\Omega_{k_o}, \Omega_b h^2, \Omega_c h^2, H_o, \omega_x$). Since the number of parameters were increased, longer chains had to be used as autocorrelation time of MCMC analysis increased. Here I've used chain length of 8000 for 200 walkers.

3.3 Comparison of Results

In the following table, I have made a comparison of results of my work with that of original paper.

Note: The paper contains only parameter values and χ_{min}^2 for the H(z)+BAO analysis but not the plots. So I've included the plots which contain analysis of H(z)+BAO+SN data (in red coloured contours in figure 3).

Attribute	Cao et al	Using emcee and scipy
$\Omega_b h^2$	0.0289	0.0295
$\Omega_c h^2$	0.0985	0.0981
Ω_m	0.296	0.296
Ω_k	-0.053	-0.055
ω_x	-0.730	-0.728
H_o	65.76	65.83
χ_{min}^2	22.13	22.12
AIC	32.13	32.12
BIC	41.05	41.04

Table 3: Unmarginalized Best Fitting Parameter Values for BAO+H(z) data

Attribute	Cao et al	Using emcee and scipy
Plots	Figure 3	Figure 4
$\Omega_b h^2$	$0.0294^{+0.0047}_{-0.0050}$	$0.0344^{+0.0040}_{-0.011}$
$\Omega_c h^2$	$0.0980^{+0.0186}_{-0.0187}$	$0.088^{+0.029}_{-0.018}$
Ω_m	0.292 ± 0.025	0.283 ± 0.029
Ω_k	-0.027 ± 0.109	-0.0282 ± 0.11
ω_x	$-0.770^{+0.149}_{-0.098}$	$-0.73^{+0.17}_{-0.11}$
H_o	$66.13^{+2.35}_{-2.36}$	$65.8^{+2.2}_{-2.5}$

Table 4: One-dimensional marginalized posterior mean values and uncertainties ($\pm 1\sigma$ error bars or 2σ limits) of the parameters

3.4 Plots

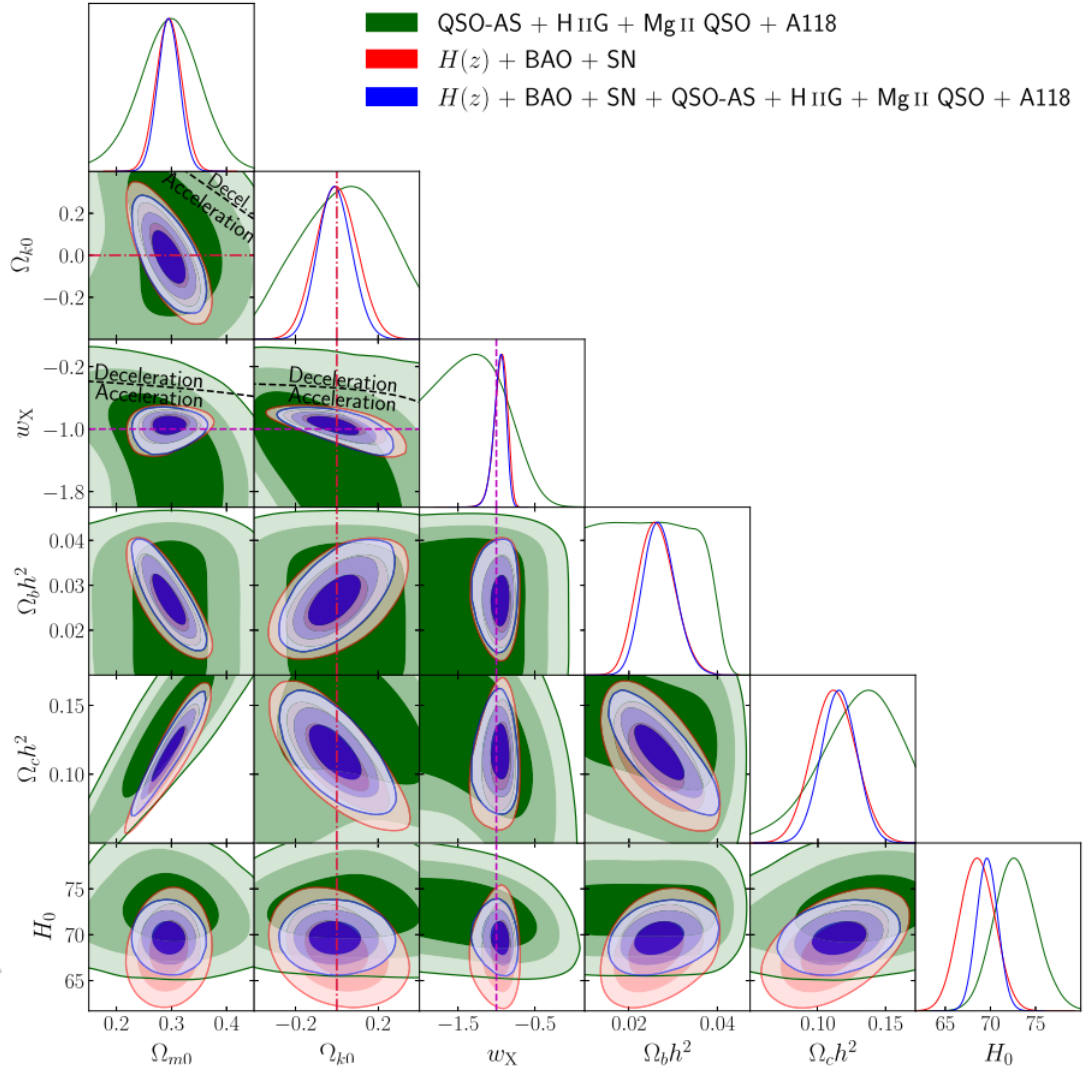


Figure 3: Original Plots in Cao et al for XCDM Model

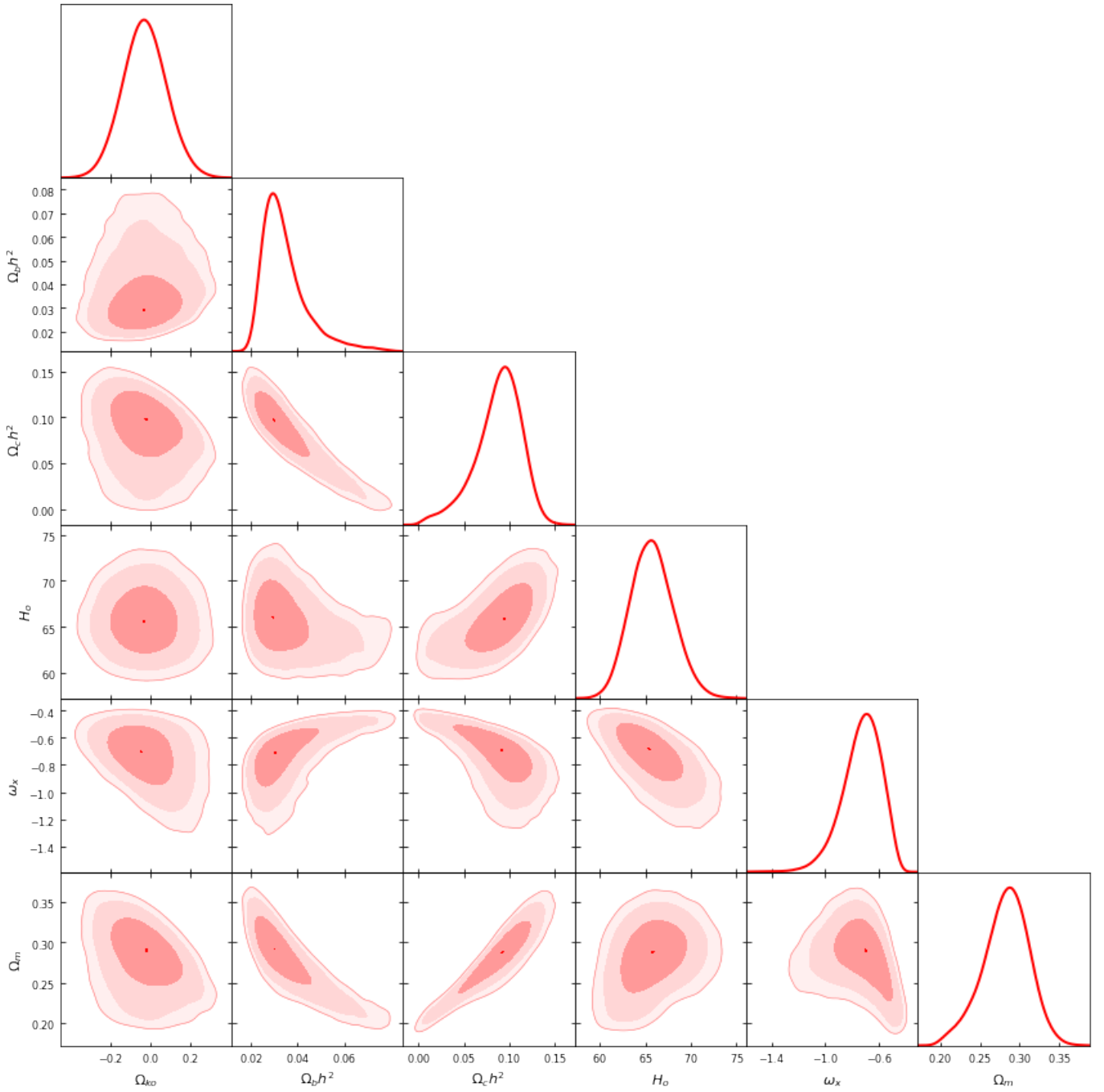


Figure 4: Plots generated using emcee and scipy for XCDM Model