# New Degrees of Freedom using $N_{eff}$ (Effective Neutrino Species) measurements of CMB

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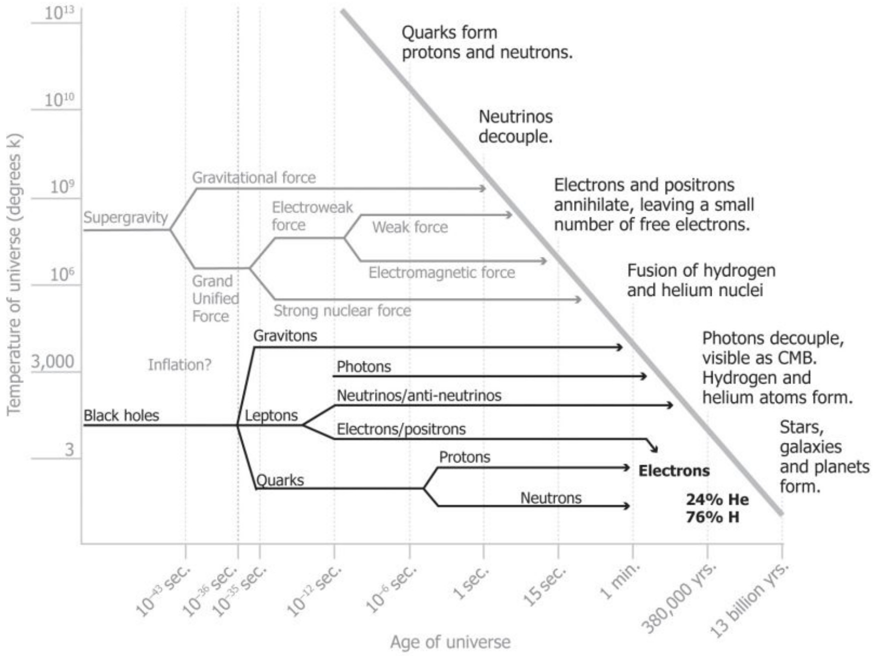
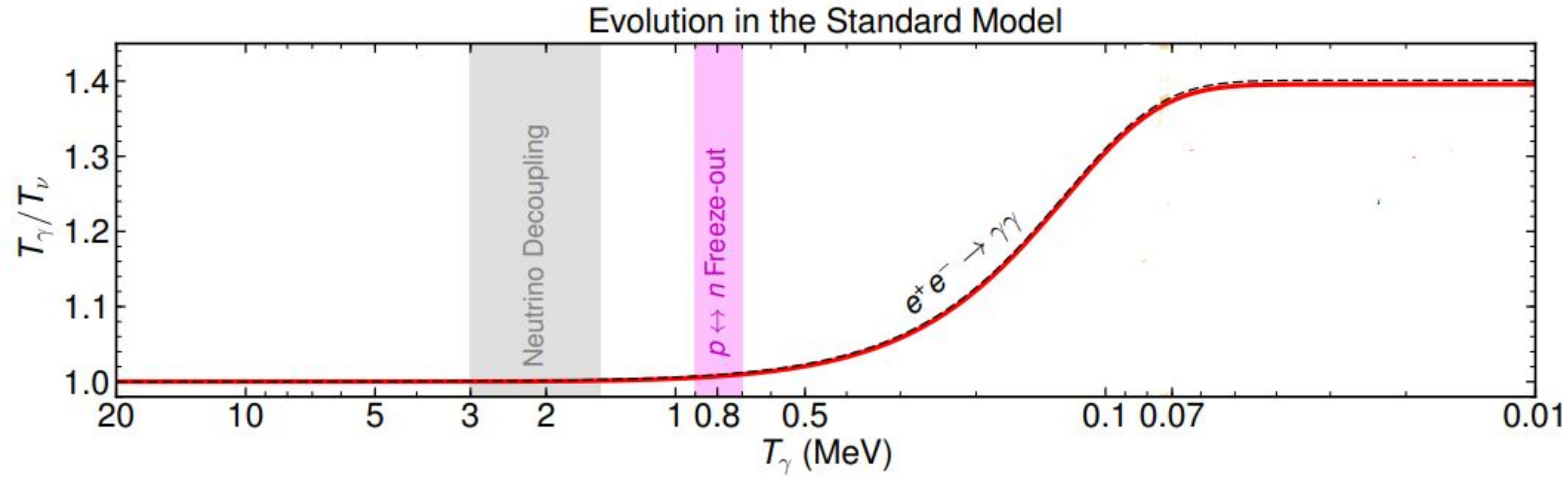
# Thermal History of the Universe



# Thermal History of the Neutrinos



# Tracing the temperature of neutrinos relative to photons

- Goal: To find the temperature ratio $T_\gamma / T_\nu$ after the $e^+e^-$ scattering
- Use entropy conservation arguments to naively estimate the ratio

$$s_i a_i^3 = s_f a_f^3$$

$$s = \frac{\rho + P}{T}$$

; s -> entropy density
i -> epoch at which neutrinos decouple
f ->epoch after the annihilations
ρ, P -> energy, pressure densities

# Tracing the temperature of neutrinos relative to photons

- For a relativistic species ρ = P/3
- At early times, particle interactions were efficient enough to keep the different species in local equilibrium. They then shared a common temperature T, and the distribution functions take the BE or FD forms

$$\rho_a = g_a \int \frac{d^3p}{2\pi^3} \frac{E(p)}{e^{(E_a-\mu_a)/T} - 1}$$

$$\rho_a = \begin{cases} \frac{\pi^2}{15} T_a^4 & \text{(boson)} \\ \frac{7}{8} \frac{\pi^2}{15} T_a^4 & \text{(fermion)} \end{cases}$$

# Tracing the temperature of neutrinos relative to photons

$$s_i = 3s_\nu + s_\gamma + s_{e^-} + s_{e^+}$$

$$s_i = \frac{4}{3}\left(3\times 2\times \frac{7}{8}\frac{\pi^2}{15}T_\nu^3 + 2\times\frac{\pi^2}{15}T_\gamma^3 + 4\times\frac{7}{8}\frac{\pi^2}{15}T_\gamma^3\right) \quad \because (T_{e^-} = T_{e^+} = T_\gamma)$$

$$s_i = \frac{4\pi^2}{45}\times\frac{43}{4}T_i^3 \quad \because T_\nu = T_\gamma = T_i$$

$$s_f = \frac{4\pi^2}{45}\left(6\times\frac{7}{8}T_\nu^3 + 2T_\gamma^3\right)$$

# Tracing the temperature of neutrinos relative to photons

$$T_i a_i = T_\nu a_f$$ (Temp of neutrinos simply dilutes by scale factor after decoupling)

$$s_i a_i^3 = s_f a_f^3 \implies T_\gamma^3 = \frac{11}{4} T_\nu^3$$

$$\rho_\nu = 3 \times \frac{7}{8} \frac{T_\nu^4}{T_\gamma^4} \rho_\gamma \implies \boxed{\rho_\nu = 3 \times \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma}$$

**Assumption:** The above analytic calculation assumes the neutrinos **instantaneously decouple** from the SM plasma at $T = T_i$. However the process of neutrino decoupling is a gradual process.

# $N_{eff}$: Effective Extra Relativistic Species

- We can define a quantity $N_{eff}$ as

$$N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_{\rm rad} - \rho_\gamma}{\rho_\gamma}$$

where $\rho_{rad}$ means total radiation density in the universe

- In the SM case radiation density is contributed by neutrinos and photons only. If the assumption of instantaneous decoupling holds, then we can see that $N_{eff} = 3$
- $N_{eff}$ serves as a convenient parameter to study the relativistic species in the early universe in terms of photon energy density

# Numerical Calculation of $N_{eff}$: Escudero's Code

$$\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = \mathscr{C}[f]$$

→ Liouville equation

- When interactions between a particle ψ and the rest of the plasma are efficient, the distribution function of the ψ species is well described by equilibrium distribution functions

- Escudero [1] showed that the relevant thermodynamic quantities can be tracked very accurately if we assume that the **species follow their equilibrium distribution at all times**

# Numerical Calculation of $N_{eff}$: Escudero's Code

Multiplying the Liouville equation by $g\frac{d^3p}{2\pi^3}$ and $g\frac{d^3p}{2\pi^3}E$ and integrating

$$\frac{dn}{dt} + 3Hn = \frac{\delta n}{\delta t} = \int g\frac{d^3p}{2\pi^3}\mathscr{C}[f]$$

$$\frac{d\rho}{dt} + 3H(\rho + \mathscr{P}) = \frac{\delta \rho}{\delta t} = \int g\frac{d^3p}{2\pi^3}\mathscr{C}[f]E$$

Using chain rule, we can formulate these differential equations in terms of Temperature T and chemical potential $\mu$

# Numerical Calculation of $N_{eff}$: Escudero's Code

$$\frac{dT}{dt} = \frac{1}{\left(\frac{\partial n}{\partial \mu}\frac{\partial \rho}{\partial T} - \frac{\partial n}{\partial T}\frac{\partial \rho}{\partial \mu}\right)}\left[-3H\left((\rho+\mathscr{P})\frac{\partial n}{\partial \mu} - n\frac{\partial \rho}{\partial \mu}\right) + \frac{\partial n}{\partial \mu}\frac{\delta \rho}{\delta t} - \frac{\partial \rho}{\partial \mu}\frac{\delta n}{\delta t}\right]$$

$$\frac{d\mu}{dt} = \frac{-1}{\left(\frac{\partial n}{\partial \mu}\frac{\partial \rho}{\partial T} - \frac{\partial n}{\partial T}\frac{\partial \rho}{\partial \mu}\right)}\left[-3H\left((\rho+\mathscr{P})\frac{\partial n}{\partial T} - n\frac{\partial \rho}{\partial T}\right) + \frac{\partial n}{\partial T}\frac{\delta \rho}{\delta t} - \frac{\partial \rho}{\partial T}\frac{\delta n}{\delta t}\right]$$

In case where the chemical potential $\mu$ can be neglected, the equations simplify further

$$\frac{dT}{dt} = \left[-3H(\rho+\mathscr{P}) + \frac{\delta \rho}{\delta t}\right] \Big/ \frac{\partial \rho}{\partial T}$$

# Neutrino decoupling in SM using Escudero's Code

- We now write down the differential equations corresponding to $T_\gamma$ and $T_\nu$ using the following three assumptions

  i. All the species involved ($e^+$,$e^-$, $\gamma$,$\nu$) follow perfect equilibrium conditions
  ii. Neutrino Oscillations are neglected: $T_\nu = T_{\nu_e} = T_{\nu_\mu} = T_{\nu_\tau}$
  iii. The chemical potentials are neglected

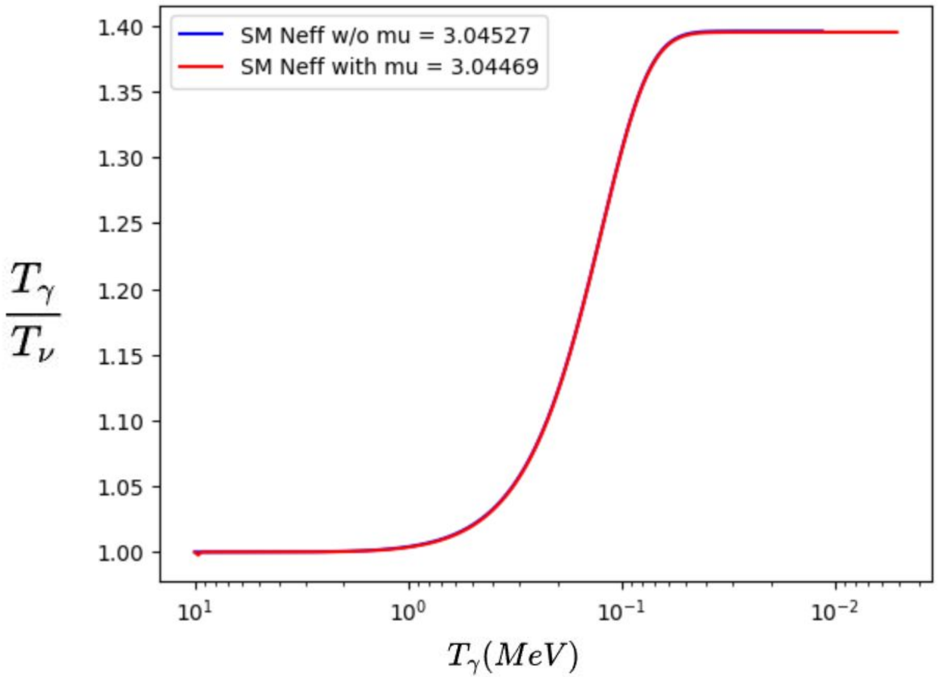
$$\frac{dT_\gamma}{dt} = -\frac{4H\rho_\gamma + 3H(\rho_e + \mathscr{P}_e) + 3HT_\gamma \frac{d\mathscr{P}_{\rm int}}{dT_\gamma} + \frac{\delta\rho_{\nu_e}}{\delta t} + 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{\frac{\partial\rho_\gamma}{\partial T_\gamma} + \frac{\partial\rho_e}{\partial T_\gamma} + T_\gamma \frac{d^2\mathscr{P}_{\rm int}}{dT_\gamma^2}}$$

$$\frac{dT_\nu}{dt} = -\frac{12H\rho_\nu - \frac{\delta\rho_{\nu_e}}{\delta t} - 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{3\frac{\partial\rho_\nu}{\partial T_\nu}}$$

FT QED Corrections

# Neutrino decoupling in SM using Escudero's Code

## Results



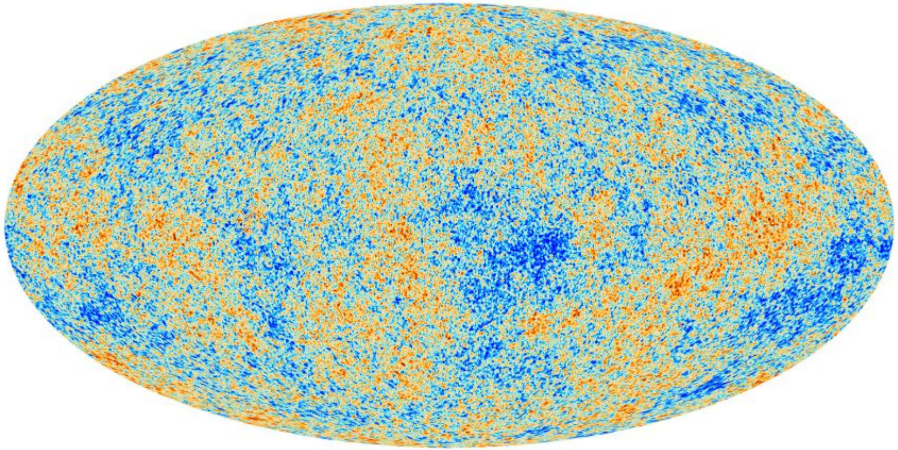
## Neutrino decoupling in SM using Escudero's Code

## Comparison

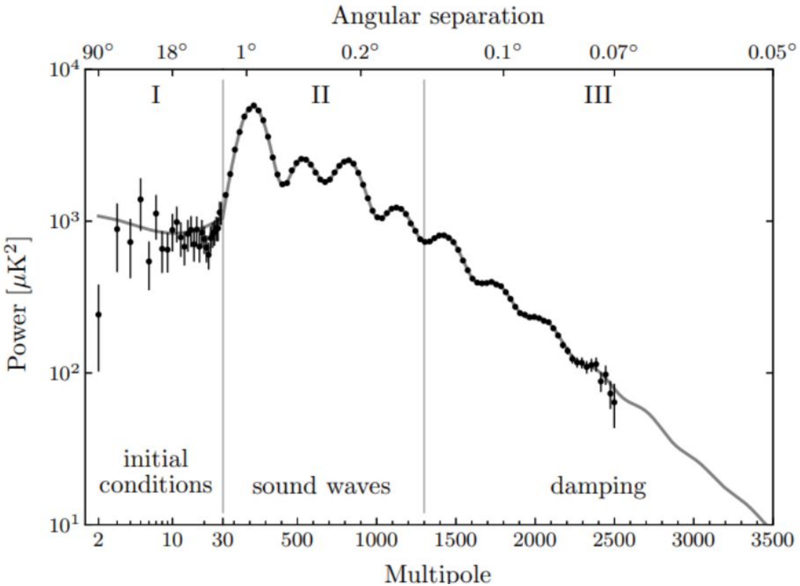
Comparison of $N_{eff}$ = 3.045 obtained using Escudero's Code with other sources

| Source/ Method | $N_{eff}$ | % change |
|---|---|---|
| Instantaneous Decoupling | 3 | 1.5 % |
| Salas Pastor et al[2] (State of the Art) | 3.046 | 0.03 % |
| Current Precision (Planck 2018) | $\sigma$ ($N_{eff}$) $\sim$ 0.2 | 5-6 % |
| Future Precision (Simons Observatory and CMB -S4) | $\sigma$ ($N_{eff}$) $\sim$ 0.05 | 1-2 % |

# From Theory to Observations: CMB Power Spectrum



Temperature fluctuations in the CMB  as measured by the Planck satellite [3]




Planck 2018 Temperature Power Spectra [4]

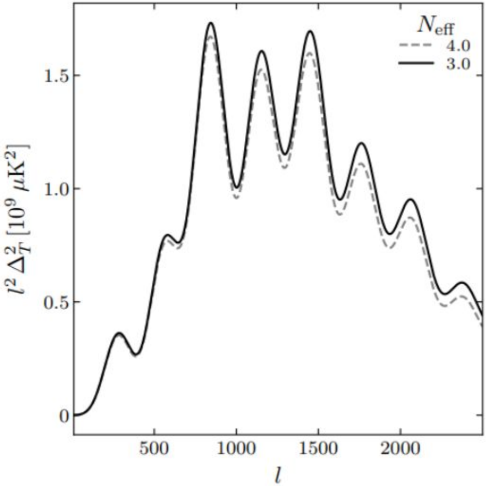
# Damping Tails

The damping tails in the region III are affected significantly (as compared to the other regions) by the amount of total radiation density before recombination.

$$\rho_{\text{rad}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma$$

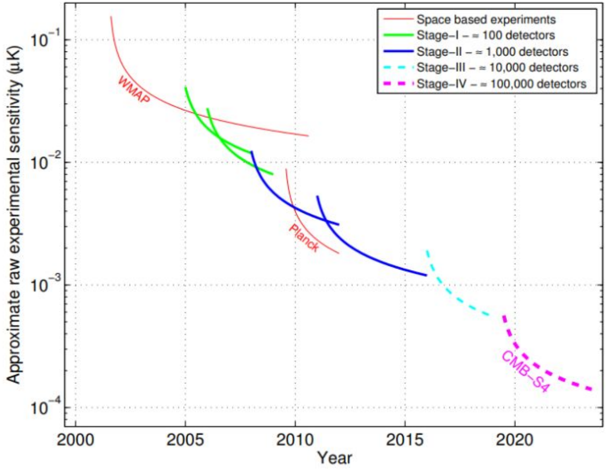
**Planck Results**

$N_{eff}$ = 2.99 ± 0.17



# CMB -S4

- The current measurement is not precise enough to conclusively determine the presence or absence of additional degrees of freedom
- CMB-S4[5] is the next-generation ground-based cosmic microwave background experiment.

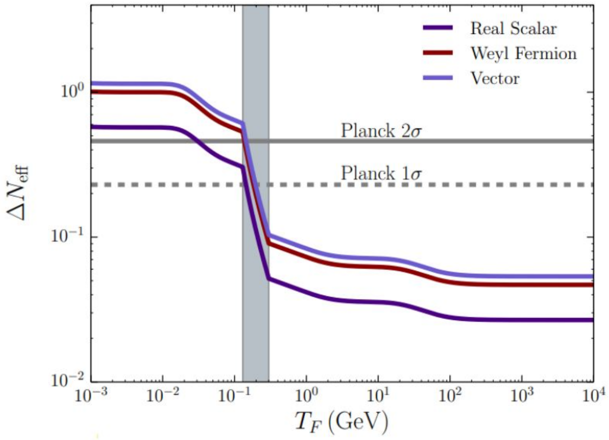


# CMB -S4 Forecasts for $N_{eff}$

- With the improved sensitivity and more precise measurements at lower angular scales, the CMB-S4 is predicted to reach $\sigma$ ($N_{eff}$ ) ~ 0.02-0.03
- For a hot thermal relic

$$\Delta N_{\rm eff} = \begin{cases} \frac{4g}{7}\left(\frac{43/4}{g_*(T_F)}\right)^{4/3} & \text{(boson)} \\ \frac{g}{2}\left(\frac{43/4}{g_*(T_F)}\right)^{4/3} & \text{(fermion)} \end{cases}$$

$\Delta N_{eff}$ > 0.047 (spin - ½. 1, 3/2)
$\Delta N_{eff}$ ~ 0.027 (spin - 0)



# BSM $N_{eff}$: Dark Matter interacting with SM via massive dark photons

## Model Description [6]

- DM particle □ (complex scalar, Majorana fermion, and Dirac fermion) coupled to a massive dark photon A' kinetically mixed with SM photon

- Equivalent Neutrino ξ (new inert, relativistic degrees of freedom)

- Model Parameters - m□ and $\Delta N_{\nu}$ ($\rho_\xi / \rho_\nu$) and mA'/m□

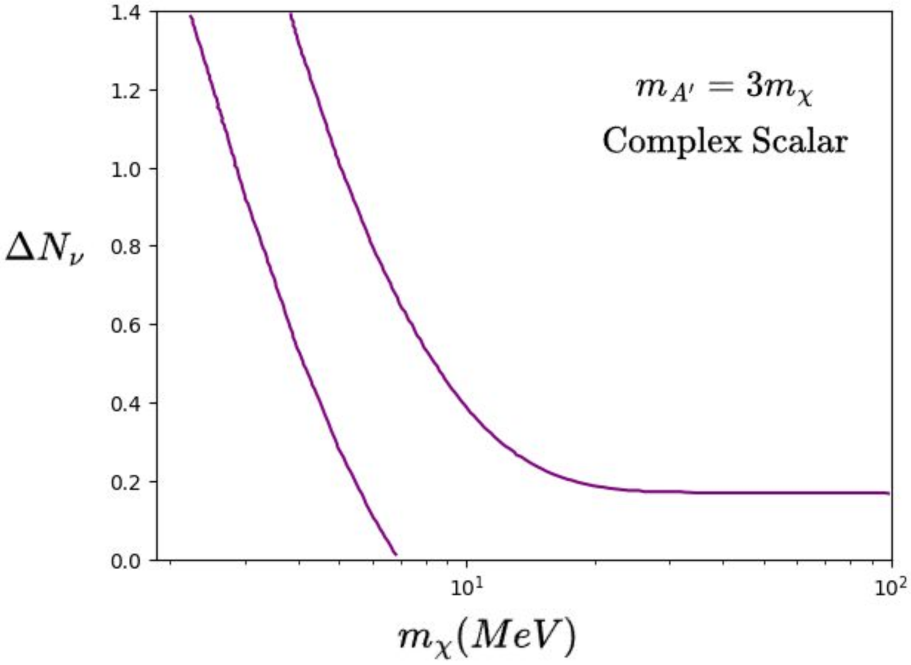
# Temperature Evolution Equations

$$\frac{dT_\gamma}{dt} = -\frac{4H\rho_\gamma + 3H(\rho_e + \mathscr{P}_e) + 3H(\rho_\chi + \mathscr{P}_\chi) + 3H(\rho_{A'} + \mathscr{P}_{A'}) + 3HT_\gamma \frac{d\mathscr{P}_{\rm int}}{dT_\gamma} + \frac{\delta\rho_{\nu_e}}{\delta t} + 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{\frac{\partial\rho_\gamma}{\partial T_\gamma} + \frac{\partial\rho_e}{\partial T_\gamma} + \frac{\partial\rho_\chi}{\partial T_\gamma} + \frac{\partial\rho_{A'}}{\partial T_\gamma} + T_\gamma \frac{d^2\mathscr{P}_{\rm int}}{dT_\gamma^2}}$$

$$\frac{dT_\nu}{dt} = -\frac{12H\rho_\nu - \frac{\delta\rho_{\nu_e}}{\delta t} - 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{3\frac{\partial\rho_\nu}{\partial T_\nu}}$$

$$\frac{dT_\xi}{dt} = -\frac{4H\rho_\xi}{\frac{\partial\rho_\xi}{\partial T_\xi}}$$

# Results




Planck $N_{eff}$ constraints on the model

- Above mχ ~ 20 MeV the predicted value of $N_{eff}$ approaches the standard cosmological value because the DM freezes out well before neutrino decoupling, and so the DM annihilations heat the electromagnetic and neutrino sectors equally.

- When mχ < 20 MeV, entropy is injected into the electromagnetic sector during and after the period of neutrino decoupling (Tvd ~ 2 MeV). In this case, $N_{eff}$ decreases relative to the standard value because the electromagnetic sector is preferentially heated. A nonzero ΔNν restores Neff to its measured value

# Future Work

- Incorporating a DM model consisting of heavy sterile right-handed neutrinos

- Sophisticated DM models involving the role of chemical potential

# References

[1] Escudero, Miguel. "Precision Early Universe Thermodynamics Made Simple:$N_{Eff}$and Neutrino Decoupling in the Standard Model and Beyond." *Journal of Cosmology and Astroparticle Physics*, vol. 2020, no. 05, May 2020, p. 048. https://doi.org/10.1088/1475-7516/2020/05/048.

[2] De Salas, Pablo F., and S. Pastor. "Relic Neutrino Decoupling With Flavour Oscillations Revisited." *Journal of Cosmology and Astroparticle Physics*, vol. 2016, no. 07, July 2016, p. 051. https://doi.org/10.1088/1475-7516/2016/07/051.

[3] Aghanim, N., et al. "*planck*2018 results." *Astronomy & Astrophysics*, vol. 641, 2020, https://doi.org/10.1051/0004-6361/201833910.

[4] Baumann, Daniel. *Cosmology*. Cambridge UP, 2022.

[5] Abazajian, Kevork N., et al. *CMB-S4 Science Book, First Edition*. 9 Oct. 2016, https://doi.org/10.2172/1352047.

[6] Giovanetti, Cara, et al. 'Joint Cosmic Microwave Background and Big Bang Nucleosynthesis Constraints on Light Dark Sectors with Dark Radiation'. *Phys. Rev. Lett.*, vol. 129, American Physical Society, July 2022, p. 021302, https://doi.org10.1103/PhysRevLett.129.021302.

# Thank You!

# Questions?