IIT BOMBAY

Krittika Summer Project

Correcting Stellar aberration using Curve Fitting

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1 Introduction

In our high school we all have come across rain problems or wind problems which involve calculating the effects of relative velocity of rain or wind wrt to an observer moving relative to them.Ever wondered such phenomena can also be possible with light rays? Although the effects won't be visible because normally the velocities we experience are very less than speed of light. But in certain case the effects of relative velocity of light are indeed observabe.For example take the case of light coming from very distant stars. Now the velocity of earth around its orbit can be roughly assumed to be 30 km/s ,which is quite high but still 0.0001 times the speed of light but with precise enough measurements we can observe these effects. Since the direction of velocity of earth also keeps on changing as it goes around its orbit the direction of relative velocity keeps on changing and thus the position of faraway star also appears to be changing.This effect known as 'Stellar Aberration' was first observed in 1725 by an english astronomer James Bradley.

2 Problem Statement

We are given the data of apparent ecliptic lattitude (β') and longitude (λ') of a star over a period of 1 year and we are required to find its true heliocentric lattitude(β) and longitude(λ).

3 Arriving at the equations

First we try to arrive at the equations of β' and λ' in terms of β and λ and then try to solve those equations to get the values of true β and λ in terms of apparent ones.

3.1 Ecliptic Coordinate System

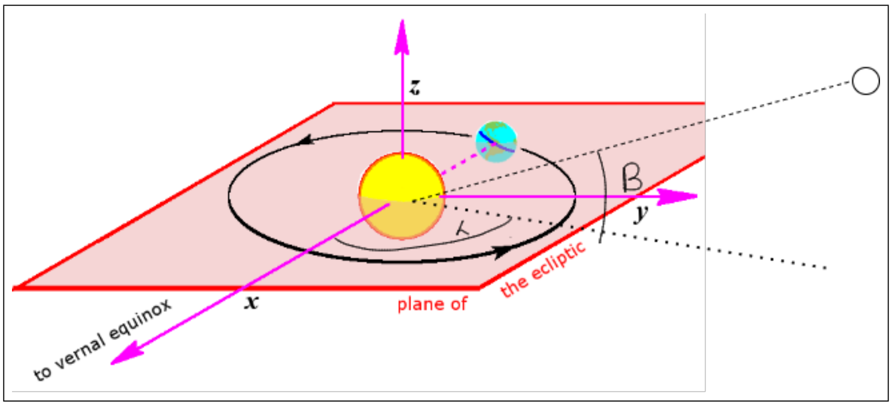

Figure 1: Ecliptic Coordinate System

The position of a distant star can be given in ecliptic cordinate system in terms of its ecliptic lattitude (β)and ecliptic longitude (λ) defined as follows :-

Ecliptic lattitude (β)-Ecliptic latitude or celestial latitude , measures the angular distance of an object from the ecliptic towards the north (positive) or south (negative) ecliptic

pole.
Ecliptic longitude (λ)-Ecliptic longitude or celestial longitude measures the angular distance of an object along the ecliptic from the primary direction.The primary direction (0° ecliptic longitude) points from the Earth towards the Sun at the vernal equinox of the Northern Hemisphere.

3.2 Relativistic Velocity Addition

Our primary task for determining stellar aberration is to calulate relative velocity of light coming from a star wrt to earth and compare the difference between the actual β and λ and apparent β and λ. But since velocity of light 'c' is involved in our calculations we should use relativistic velocity addition formulae for getting more accurate results.
For an observer moving with velocity v along x axis relative velocity of a particle moving with velocities v_x, v_y, v_y in x,y,z direction respectively is given by

$$v'_x = \frac{v_x - v}{1 - \frac{v.v_x}{c^2}} \tag{1}$$

$$v'_y = \frac{\sqrt{1 - \frac{v^2}{c^2}}.v_y}{1 - \frac{v.v_x}{c^2}} \tag{2}$$

$$v'_z = \frac{\sqrt{1 - \frac{v^2}{c^2}}.v_z}{1 - \frac{v.v_x}{c^2}} \tag{3}$$

3.3 Expressions for apparent $\beta(\beta')$ and apparent $\lambda(\lambda')$

Since the formulae for relativistic velocity addition are defined for an observer moving along x direction we'll also assume that the direction of velocity of earth at any particular instant is along x direction.
Since the star is located far away from the sun the latitude angle β can be assumed to be same for both earth and sun to be same as β.
Also as the earth moves around its orbit, the angle in xy plane(ecliptic plane) that light makes with the direction of motion of earth also keeps on changing.
It can be easily shown that the angle in xy plane that light coming from a star located at ecliptic longitude λ makes with the direction of motion of earth at nth day from vernal equinox is given by

$$\phi = 90 + \lambda - \frac{360n}{365} \tag{4}$$

Now that we have got angles that light coming from star makes with xy plane (β) and x-axis(ϕ) we can break velocity of light into its x,y,z components as

$$c_z = c \sin \beta \tag{5}$$

$$c_y = -c \cos \beta \sin \phi \tag{6}$$

$$c_x = -c \cos \beta \cos \phi \tag{7}$$

Assuming earth moves in a circular orbit of radius 1.496×10^8 km of time period 365 days the velocity of earth can be calculated as 29.783 km/s. For now we take this velocity to be 'v'. For making equations look a bit clean let us write $\frac{v}{c}$ as α and $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ as γ.
We can write equations of relative velocity of light wrt earth as :-

$$c'_x = c\left(\frac{-\cos\beta\cos\phi - \alpha}{1+\alpha\cos\beta\cos\phi}\right) \tag{8}$$

$$c'_y = c\left(\frac{-\gamma\cos\beta\sin\phi}{1+\alpha\cos\beta\cos\phi}\right) \tag{9}$$

$$c'_z = c\left(\frac{\gamma\sin\beta}{1+\alpha\cos\beta\cos\phi}\right) \tag{10}$$

From 2nd postulate of special relativity we know that the relative velocity of light c' in earth's frame(or in any inertial frame) is c. Now we calculate apparent angles β' and ϕ'.

$$\sin\beta' = \frac{c'_z}{c'} = \frac{c'_z}{c}$$

$$\boxed{\sin\beta' = \frac{\sin\beta}{1+\alpha\cos\beta\cos\phi}} \tag{11}$$

$$\tan\phi' = \frac{c'_y}{c'_x}$$

$$\boxed{\tan\phi' = \frac{\gamma\cos\beta\sin\phi}{\cos\beta\cos\phi+\alpha}} \tag{12}$$

Once we have obtained ϕ' ,λ' can be obtained as

$$\boxed{\lambda' = \phi' - 90 + \frac{360n}{365}}$$

4 Solving Equations

Looking at the form of equations 11 and 12 seems like it is a quite painful task to solve them by hand ,that's where Python(or any other computational tool) comes in the picture. We'll use curve fitting technique to use the given dataset and above equations to obtain the values of true β and λ of the star.Refer this article to find out more about curve fitting tool.
Link for the project repository containing datset and .ipynb file.

5 Results

Apparent β v/s λ was plotted for the star over the period of year and also the values of actual coordinates was marked as shown.

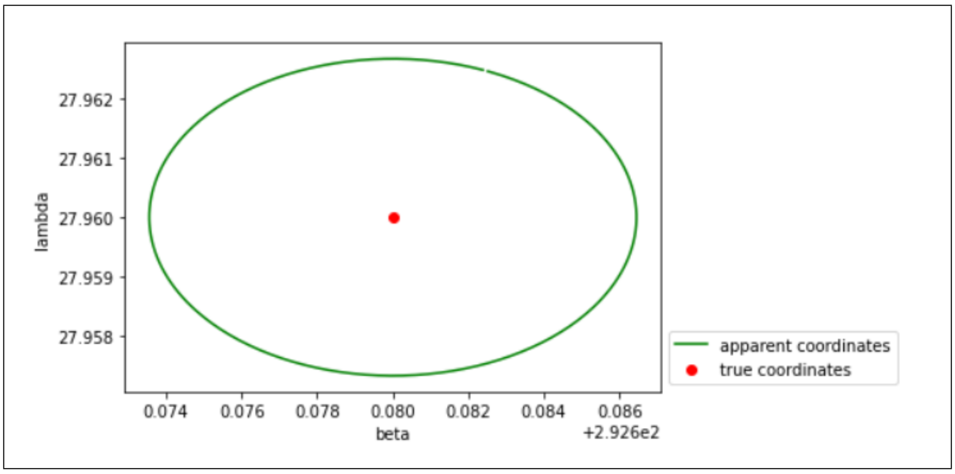

Figure 2: β v/s λ curve

To get a feel of how the trajectory of the star looks like I tried plotting a 3d graph showing the actual observed trajectory of the star. Since we are given that the star is very far away so for visulaisation I took the distance of star equal to that of Alpha Centauri i.e. 4.37 light years. Using β as θ and λ as ϕ in spherical polar coordinates we can get cartesian coordinates of a star and they can be plotted to give a curve like as shown below.

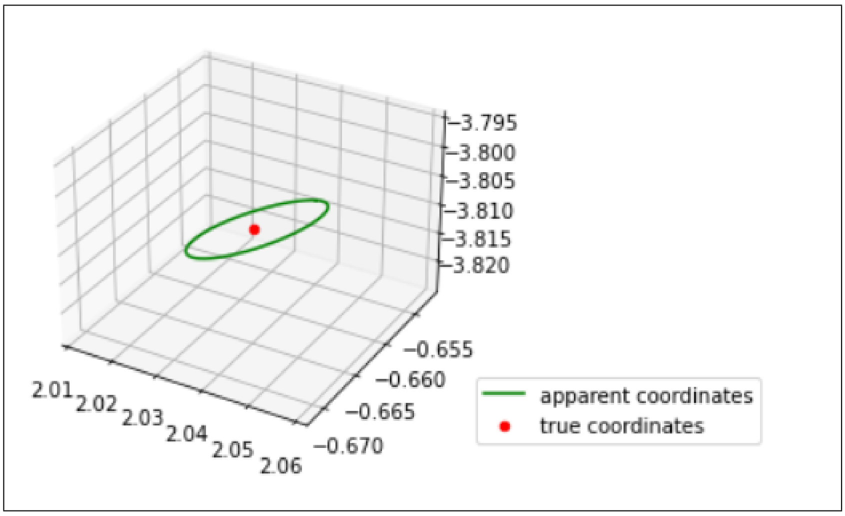

Figure 3: Trajectory of star as observed by an observer on earth