

Parameter Constraints

Prakhar Bansal

June 2022

1 Introduction

I have reproduced the plots given in the original paper Farooq et al for Λ CDM and XCDM models. For each model the contour plots are generated in two ways. First method involves the use of Matplotlib's standard library function to plot contour diagrams while the second method involves the use of MCMC simulation to obtain Markov Chains for each free parameter and using those to generate plots. In the following sections I've first described both methods in detail and compared the results of my work with that of actual paper and shown the plots made by me and present in the original paper.

2 Methods

2.1 Method 1: Using Matplotlib's standard library function

First I wrote the code for χ^2 function and then minimized it using `scipy.minimize`. It returns the minimized value of function χ_{min}^2 and the best fit value of free parameters Ω_{mo} and Ω_{Λ} (ω_x in XCDM model) which minimizes χ^2 . After getting the best fit parameters, to plot 1σ 2σ and 3σ contours I used matplotlib's standard library function `matplotlib.pyplot.subplots.contour` and plotted the values of Ω_{mo} and Ω_{Λ} corresponding to $\chi_{min}^2 + 2.3$, $\chi_{min}^2 + 6.17$, $\chi_{min}^2 + 11.8$. The best fit parameters were used to calculate the z_{da} value, the deceleration-acceleration transition redshift.

2.2 Method 2: Using MCMC

In this method, the first half of the procedure remains the same as previous method. The function χ^2 is minimized using `scipy.minimize` and corresponding minimum value χ_{min}^2 and best parameters are obtained. After this MCMC simulations were run to obtain Markov Chains for each parameter. The chain length was chosen as 10000 and no. of walkers as 200. These markov chains were used to calculate marginalized distribution for each parameter which in turn were used to obtain 2σ bounds for each parameter. The markov chains were also used to generate 1σ 2σ and 3σ contours.

3 Λ CDM Model

3.1 Comparison of Results

In the following table, I have made a comparison of results of my work with that of original paper. All the results were computed for two values of H_o viz $H_o \pm \sigma_{H_o} = 68 \pm 2.8 \text{ km s}^{-1}$ and $H_o \pm \sigma_{H_o} = 73.8 \pm 2.4 \text{ km s}^{-1}$. The values in square brackets ([]) indicate the results corresponding to $H_o \pm \sigma_{H_o} = 73.8 \pm 2.4 \text{ km s}^{-1}$.

Note: For both methods the attributes Best Fit Point, χ_{min}^2 , z_{da} are same because these values only depend on the function used to minimize χ^2 which was same (`scipy.minimize`) for both.

Note: The two methods differ in the nature of plots they generate and the values of 2σ bound they give for each parameter. Since the first method doesn't generate markov chains, we can't calculate marginalized distributions for each parameter (and hence 2σ bounds) using first method.

Attribute	Farooq et al	Method 1(w/o MCMC)	Method 2(using MCMC)
Plots	Figure 1	Figure 2	Figure 3
Best Fit Point (Ω_m, ω_x)	(0.29,0.72) [(0.32,0.91)]	(0.29,0.72) [(0.32,0.91)]	(0.29,0.72) [(0.32,0.91)]
χ^2_{min}	(18.24) [19.30]	(18.07) [19.1]	(18.07) [19.1]
2σ intervals	$0.15 \leq \Omega_{m0} \leq 0.42$ [$0.20 \leq \Omega_{m0} \leq 0.44$] $0.35 \leq \omega_x \leq 1.02$ [$0.62 \leq \omega_x \leq 1.14$]	-	$0.15 \leq \Omega_{m0} \leq 0.43$ [$0.19 \leq \Omega_{m0} \leq 0.44$] $0.33 \leq \omega_x \leq 1.01$ [$0.60 \leq \omega_x \leq 1.15$]
z_{da}	(0.706) [0.785]	(0.705) [0.784]	(0.705) [0.784]

3.2 Plots

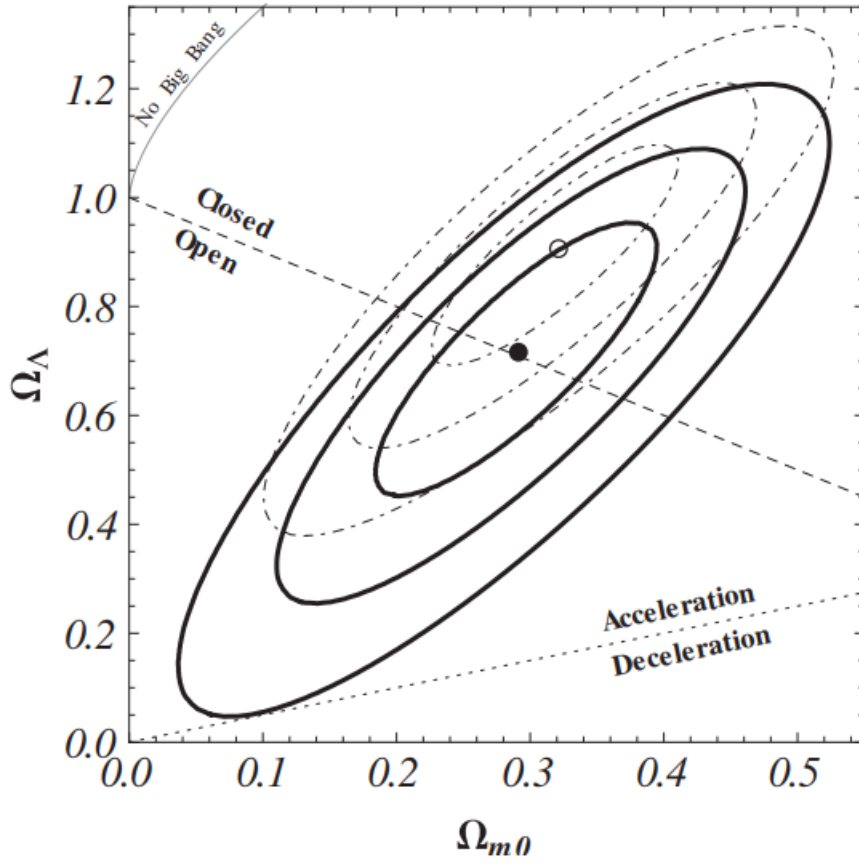


Figure 1: Original Plot in Farooq et al

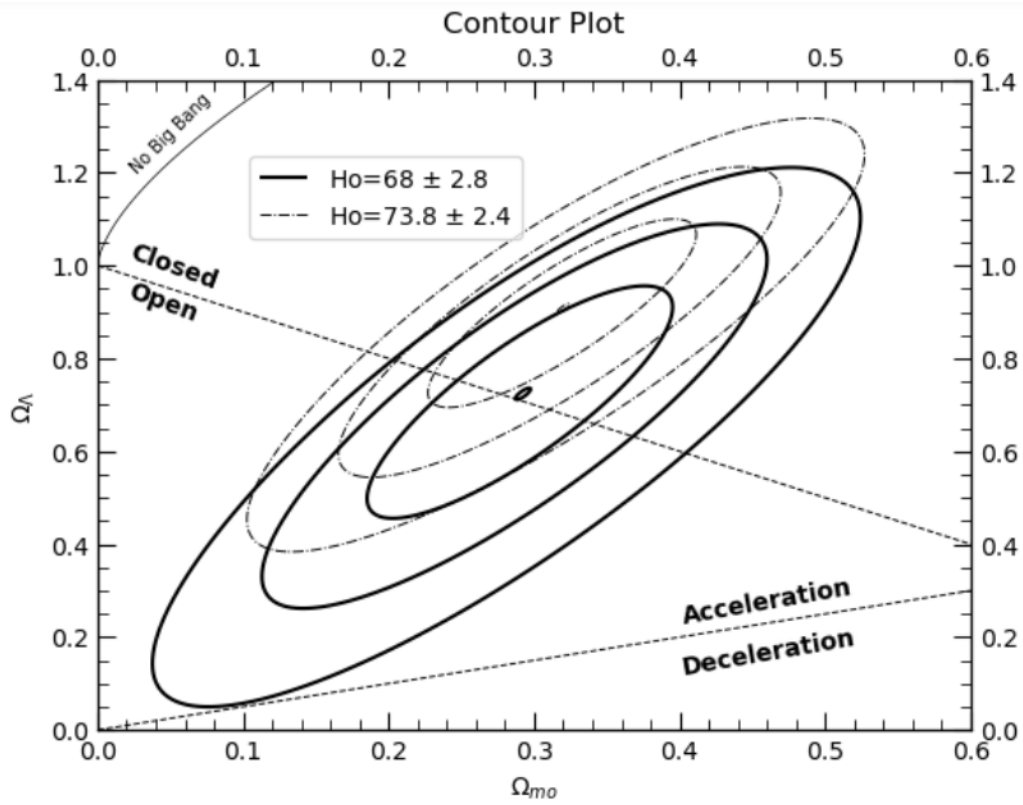


Figure 2: Plot generated using Matplotlib std library function

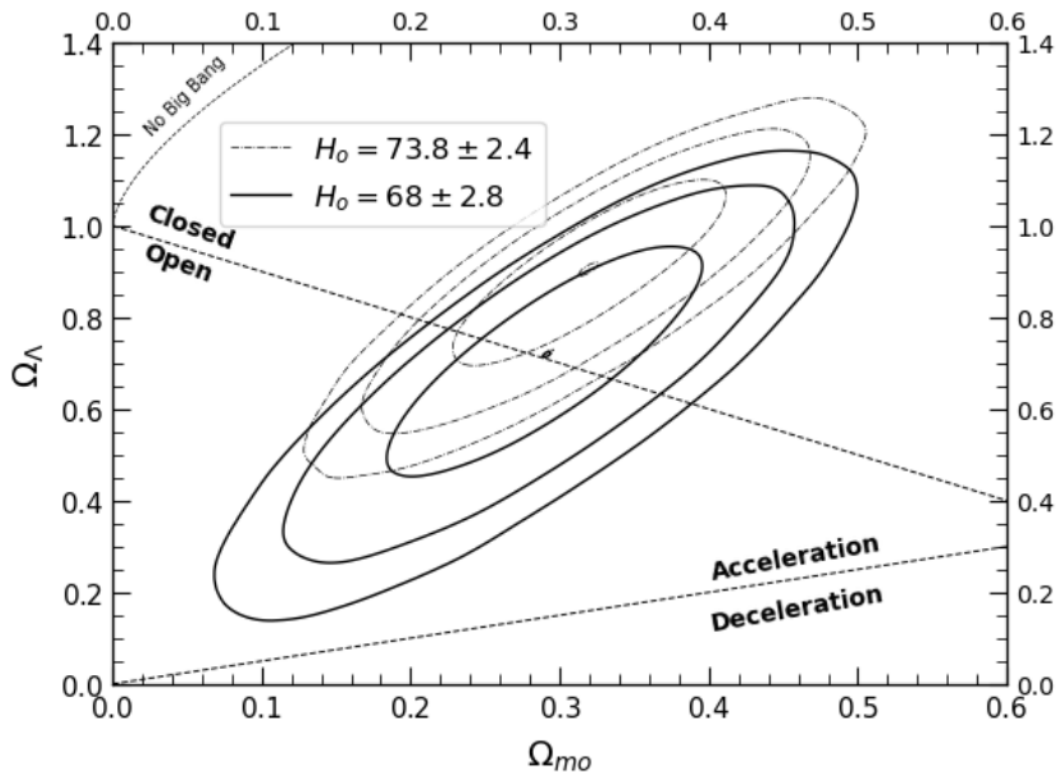


Figure 3: Plot generated using MCMC method

4 XCDM Model

4.1 Comparison of Results

Attribute	Farooq et al	Method 1(w/o MCMC)	Method 2(using MCMC)
Plots	Figure 4	Figure 5	Figure 6
Best Fit Point (Ω_{mo}, ω_x)	(0.29,-1.04) [(0.26,-1.30)]	(0.29,-1.05) [(0.26,-1.31)]	(0.29,-1.05) [(0.26,-1.05)]
χ^2_{min}	(18.18) [18.15]	(18.02) [17.92]	(18.02) [17.92]
2σ intervals	$0.23 \leq \Omega_{mo} \leq 0.35$ $[0.22 \leq \Omega_{mo} \leq 0.31]$ $-1.51 \leq \omega_x \leq -0.64$ $[-1.78 \leq \omega_x \leq -0.92]$	-	$0.23 \leq \Omega_{mo} \leq 0.36$ $[0.22 \leq \Omega_{mo} \leq 0.32]$ $-1.54 \leq \omega_x \leq -0.65$ $[-1.82 \leq \omega_x \leq -0.94]$
z_{da}	(0.695) [0.718]	(0.695) [0.705]	(0.695) [0.705]

4.2 Plots

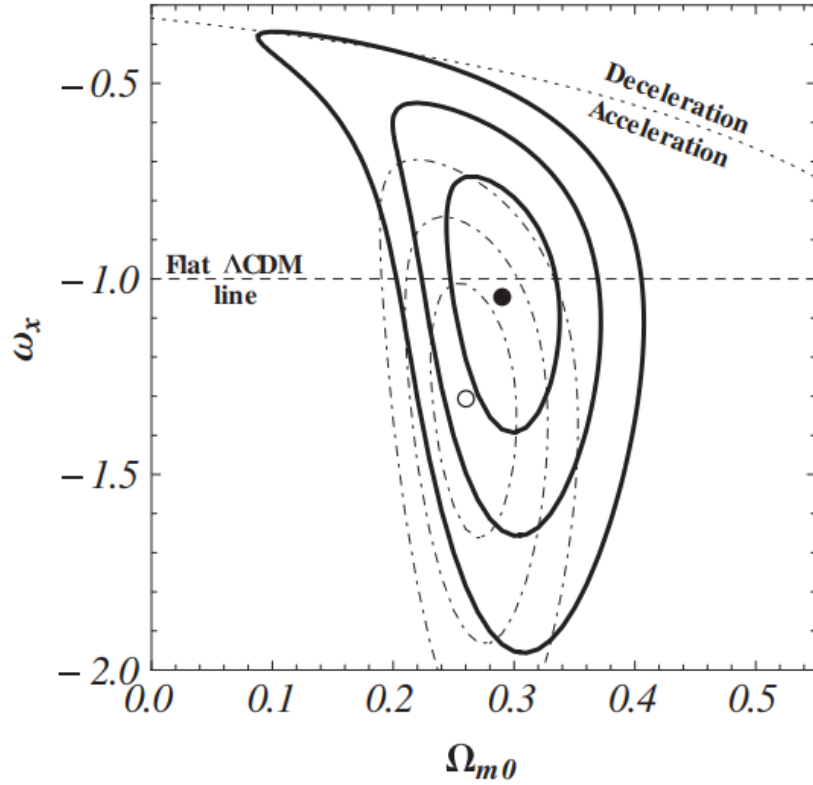


Figure 4: Original Plot in Farooq et al

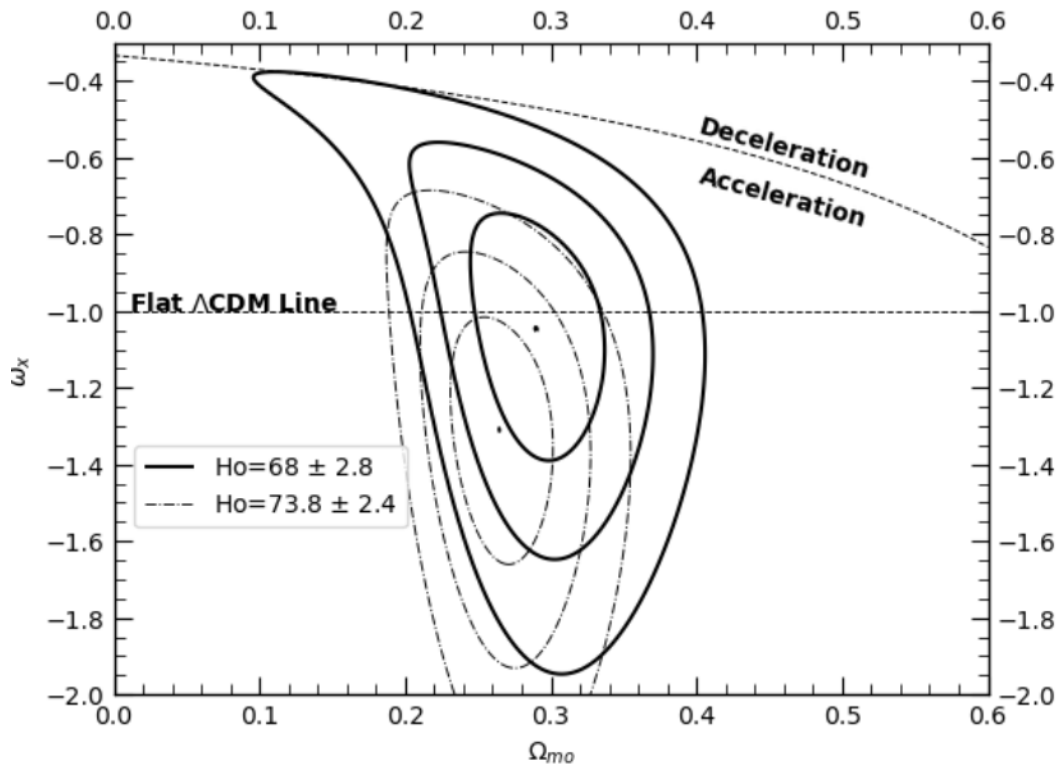


Figure 5: Plot generated using Matplotlib std library function

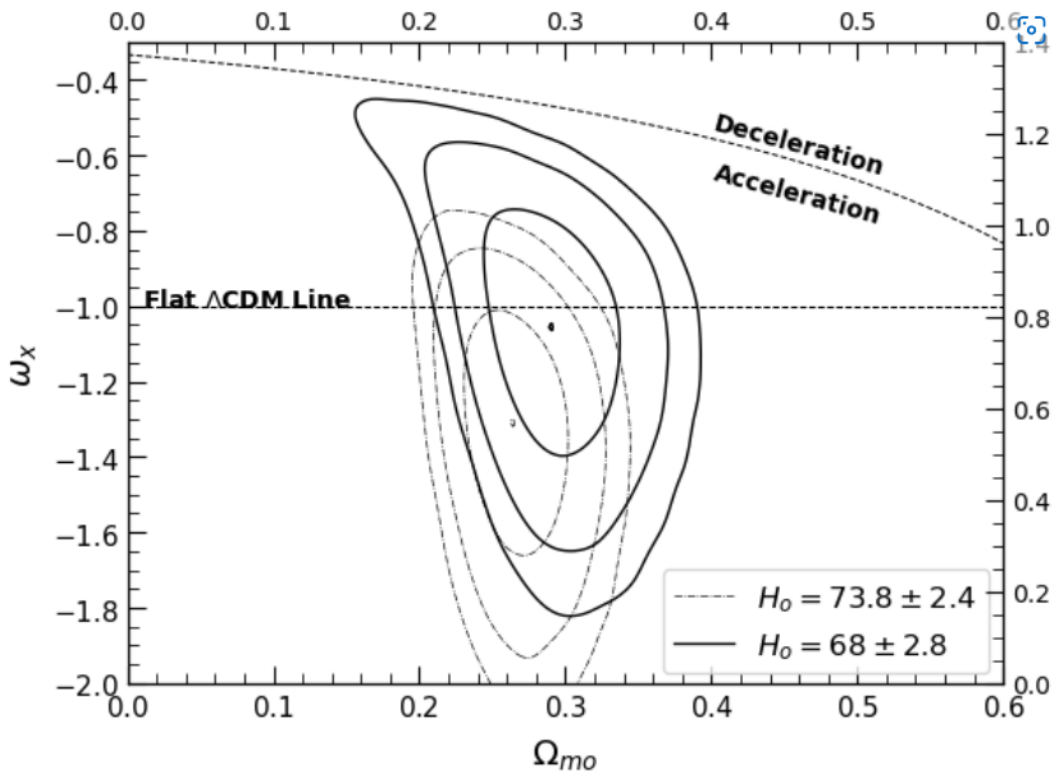


Figure 6: Plot generated using MCMC method