Basics of 21-cm Cosmology

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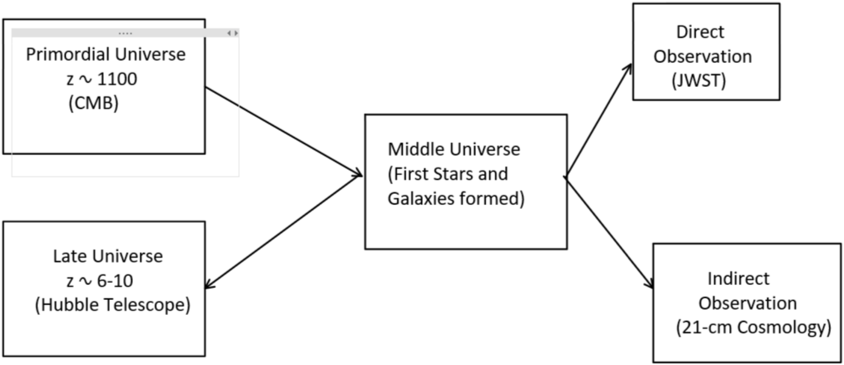
Outline

- Intoduction
- Radiative Transfer Equations
- Spin temperature of H atom
- Optical Depth
- Global 21-cm Signal
- Summary

Introduction



Brightness Temperature

- Planck's law gives us the specific intensity distribution of a blackbody in terms of the frequency (or wavelength) of the radiation emitted by the body

$$I_s(\nu) = \frac{2h\nu^3}{c^2}.\frac{1}{e^{\frac{h\nu}{k_b T}} - 1} \tag{1}$$

- In the limit $h\nu << k_b T$ the above relation reduces to

$$I_s(\nu) \approx \frac{2h\nu^3}{c^2}.\frac{1}{1 + \frac{h\nu}{k_b T} - 1} \tag{2}$$

$$= \frac{2k_b T\nu^2}{c^2} \tag{3}$$

This limit is known as Rayleigh Jeans limit of small frequencies

- This temperature T is used as a proxy for the specific intensity and is termed as the Brightness Temperature of the radiation

Spin Temperature

- The opposite and same spins of the proton and the electron in a neutral H atom interact and give rise to a singlet and triplet states respectively as shown

$$|00\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{4}$$

$$|1,-1\rangle = |\downarrow\downarrow\rangle, |1,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), |1,1\rangle = |\uparrow\uparrow\rangle \tag{5}$$

- The energy difference between the two state is $\Delta = 5.9\mu eV$ correspond to a photon of 21 cm wavelength
- In a sample of neutral H at equilibrium, the ratio of number of atoms in triplet to singlet state is given by

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{\Delta}{k_B T_S}} = 3e^{-\frac{\Delta}{k_B T_S}} \tag{6}$$

where T_S is a characteristic temprature of the sample known as Spin Temperature

Radiative Transfer Equations

- In cosmological contexts the 21 cm line has been used as a probe of gas along the line of sight to some background radio source.

- The basic equation of radiative transfer for the specific intensity I_ν ($\frac{dI}{d\nu d\Omega}$)in the absence of scattering along a path described by coordinate s

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \tag{7}$$

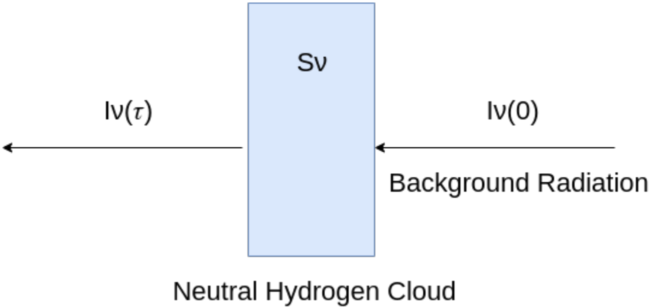
  where absorption and emission by gas along the path are described by the coefficients α_ν and j_ν, respectively.

- Using the standard definition of optical depth $d\tau_\nu = \alpha ds$ and writing $j_\nu/\alpha = S_\nu$ we obtain

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \tag{8}$$

  S_ν is the intensity emitted by the source

Radiative Transfer Equations



- Background Radiation
  - CMB - In this case, $I_\nu = B_\nu(T_{CMB})$ and the 21 cm feature is seen as a spectral distortion to the CMB blackbody at appropriate radio frequencies
  - Radio Loud Point Source - Eg Quasar

Radiative Transfer Equations

- Source Term- Since CMB have energies in radio regime they can interact with neutral H only through the hyperfine transition, hence the radiation from the source which is of primary interest to us is the one having characteristic brightness temperature as spin temperature

$$S_\nu = B_\nu(T_S) = \frac{2k_b\nu^2 T_S}{c^2} \tag{9}$$

- To simplify the expressions, we work in Rayleigh Jeans limit. Substituting (3) in (5) and cancelling $\frac{2k_b\nu^2}{c^2}$ from both sides we get

$$\frac{dT_B}{d\tau_\nu} = -T_B + T_S$$

$$\frac{dT_B}{d\tau_\nu} e^{\tau_\nu} + T_B e^{\tau_\nu} = T_S e^{\tau_\nu} \tag{10}$$

Radiative Transfer Equations

$$\int_0^{\tau_\nu} \left(\frac{dT_B}{d\tau_\nu} e^{\tau_\nu} + T_B e^{\tau_\nu} \right) d\tau_\nu = \int_0^{\tau_\nu} T_S e^{\tau_\nu} d\tau_\nu$$

$$\left(T_B e^{\tau_\nu} \right) \Big|_0^{\tau_\nu} = T_S \left(e^{\tau_\nu} \right) \Big|_0^{\tau_\nu}$$

$$T_B(\tau_\nu) e^{\tau_\nu} - T_B(0) = T_S(e^{\tau_\nu} - 1)$$

$$\boxed{T_B(\tau_\nu) = T_B(0) e^{-\tau_\nu} + T_S(1 - e^{-\tau_\nu})} \tag{11}$$

T_B- Brightness temperature of background radio source

T_S- Spin Temperature of Neutral H Cloud

Radiative Transfer Equations

- In case of an optically thin cloud ($\tau_\nu << 1$)

$$T_B(\tau_\nu) - T_B(0) = -T_B(0)\tau_\nu + T_S\tau_\nu \tag{12}$$

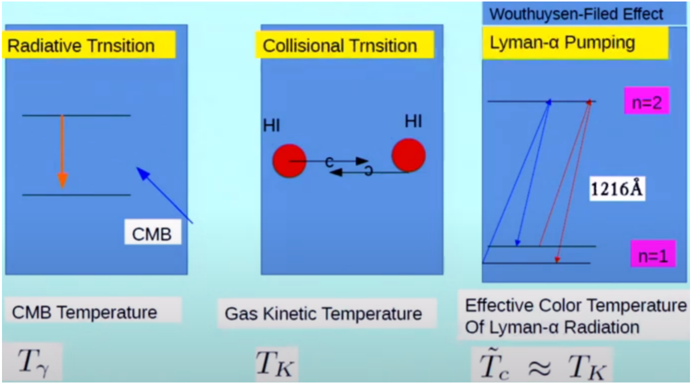
If the cloud is present at a redshift z, the temperatures get redshifted by (1+z)

$$\boxed{\delta T_B = \frac{T_S - T_B}{1+z}\tau_\nu} \tag{13}$$

Factors affecting Spin Temperature

- The key to the detectability of the 21 cm signal hinges on the spin temperature. Only if this temperature deviates from the background temperature, will a signal be observable.
- Three processes determine the spin temperature:
  1. Absorption/emission of 21 cm photons from/to the radio background, primarily the CMB;
  2. Collisions with other hydrogen atoms and with electrons;
  3. Resonant scattering of Lyα photons that cause a spin flip via an intermediate excited state

Excitation De-excitation



- Each of the above process tries to bring the spin temperature equal to its characteristic temperature T_γ, T_K, $\tilde{T}_C$

Rate Equation

$$\dot{n}_0 = -(P_{01}^{\gamma} + P_{01}^{C} + P_{01}^{\alpha})n_0 + (P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha})n_1 \tag{14}$$

- Detailed Balance
  At equilibrium, each elementary process is in equilibrium with its reverse process. Consider a situation where radiative tranistion (due to CMB) dominate ($T_S = T_\gamma$). At equilibrium

$$\dot{n}_0 = 0$$

$$n_0 P_{01}^{\gamma} = n_1 P_{10}^{\gamma}$$

$$\frac{P_{01}^{\gamma}}{P_{10}^{\gamma}} = \frac{n_1}{n_0} = 3e^{-\frac{T_*}{T_\gamma}}$$

$$\boxed{\frac{P_{01}^{\gamma}}{P_{10}^{\gamma}} \approx 3(1 - \frac{T_*}{T_\gamma})} \tag{15}$$

where $T_* = \Delta / k_B$

Rate Equation

We can obtain similar relations for rate constants corresponding to Collisional Coupling and Lyα Coupling

$$\boxed{\frac{P_{01}^{C}}{P_{10}^{C}} \approx 3(1 - \frac{T_*}{T_K})} \tag{16}$$

$$\boxed{\frac{P_{01}^{\alpha}}{P_{10}^{\alpha}} \approx 3(1 - \frac{T_*}{T_C})} \tag{17}$$

Applying equilibrium condition in (15), we obtain

$$\frac{n_1}{n_0} = \frac{P_{01}^{\gamma} + P_{01}^{C} + P_{01}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}}$$

$$3(1 - \frac{T_*}{T_S}) = \frac{3(1 - \frac{T_*}{T_\gamma})P_{10}^{\gamma} + 3(1 - \frac{T_*}{T_K})P_{10}^{C} + 3(1 - \frac{T_*}{T_C})P_{10}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}} \tag{18}$$

Rate Equation

$$T_S^{-1} = \frac{T_\gamma^{-1} P_{10}^{\gamma} + {T_K}^{-1} P_{10}^{C} + {T_C}^{-1} P_{10}^{\alpha}}{P_{10}^{\gamma} + P_{10}^{C} + P_{10}^{\alpha}}$$

$$\boxed{T_S^{-1} = \frac{T_\gamma^{-1} + T_K^{-1} x_C + T_C^{-1} x_\alpha}{1 + x_C + x_\alpha}} \tag{19}$$

$$x_C = \frac{P_{10}^{C}}{P_{10}^{\gamma}}$$

$$x_\alpha = \frac{P_{10}^{\alpha}}{P_{10}^{\gamma}}$$

Coupling Coefficients

- Collisional Coupling Three main channels are available -:
  1. collisions between two hydrogen atoms
  2. collisions between a hydrogen atom and an electron or
  3. a proton

$$x_C = x_C^{HH} + x_C^{eH} + x_C^{pH}$$

$$\boxed{x_C = \frac{T_*}{A_{10}\,T_\gamma}\left(\kappa_{1-0}^{HH}(T_k)n_H + \kappa_{1-0}^{eH}(T_k)n_e + \kappa_{1-0}^{pH}(T_k)n_p\right)}$$

where κ_{1-0}^{Hi} is the scattering rate for the collision with i th species and A_{10} is the Einstein coefficient for Spontaneous Emission

Coupling Coefficients

- Lyman α Coupling

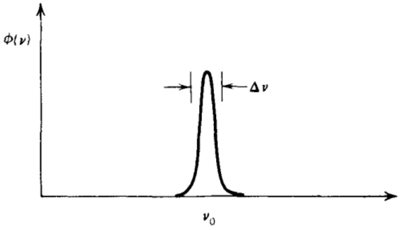
$$\boxed{x_\alpha = \frac{S_\alpha J_\alpha}{1.165 \times 10^{10}[(1+z)/20] cm^{-2} s^{-1} Hz^{-1} sr^{-1}}} \quad (20)$$

where S_α is a factor of order unity and J_α is specific flux $(dN/dAdtd\nu d\Omega)$ of Lymanα photons

Line Profile

- Owing to Heisenberg's uncertainity relation the energy difference between the two levels is not infinitely sharp but is described by a line profile function $\phi(\nu)$ which is sharply peaked at $\nu = \nu_o$ and which is conveniently taken to be normalized

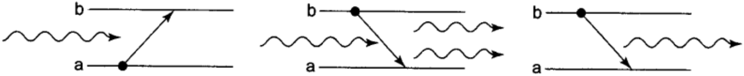
$$\int \phi(\nu) d\nu = 1$$



- This line profile function describes the relative effectiveness of frequencies in the neighborhood of ν_o for causing transitions.

Einstein's Coefficients

- The relationship between macroscopic absorption and emission coefficients α_ν and j_ν (7) and absorption and emission at microscopic level was first studied by Einstein. Three processes were identified
  1. Spontaneous Emission
  2. Absorption
  3. Stimulated Emission



(a) Absorption (b) Stimulated emission (c) Spontaneous emission

- Spontaneous Emission Coefficient
  $A_{10} =$ transition probability per unit time for spontaneous emission(s^{-1})

Einstein's Coefficients

- Absorption Coefficient Since the transition occours in the presence of photons of energy $h\nu_o$(and nearby), we expect that the probability per unit time for this process will be proportional to the density of photons (or to the mean intensity) at frequency ν_o
  We express the transition probability per unit time as $B_{01}\bar{J}$ where
  $$\bar{J} = \int J_\nu \phi(\nu) d\nu \qquad J_\nu = \int \frac{I_\nu}{4\pi} d\Omega$$
  For an isotropic radiation $J_\nu = I_\nu$
  In case of most of the radiations J_ν changes slowly over the width $\Delta\nu$ of the line, $\phi(\nu)$ behaves like a δ function thus for an isotropic such radiation rate of transition probabilitiy is simply $B_{01} I_\nu$
- Stimulated Emission Coefficient
  Rate of Transition Probability $= B_{10} I_\nu$

Relation between Einstein Coefficients

- In thermal equilibrium

$$n_1 A_{10} + n_1 B_{10} I_\nu = n_0 B_{01} I_\nu$$

$$I_\nu = \frac{A_{10}/B_{10}}{(n_0 B_{01})/(n_1 B_{10}) - 1}$$

$$I_\nu = \frac{A_{10}/B_{10}}{(g_0 B_{01}/g_1 B_{10}) e^{\frac{\Delta}{k_B T_S}} - 1}$$

n_1, n_0 is the number density of atoms in respective energy levels

Comparing with (1) we get

$$A_{10} = \frac{2h\nu^3}{c^2} B_{10} \tag{21}$$

$$g_0 B_{01} = g_1 B_{10} \tag{22}$$

Connection with Macroscopic Coefficients

- Emission Coefficient j_ν
  The amount of energy emitted in volume dV, solid angle $d\Omega$, frequency range $d\nu$, and time dt is, by definition, $j_\nu dV d\Omega d\nu dt$
  Each atom contributes an energy $h\nu_o$, distributed over 4π solid angle for each transition, this may also be expressed as $(h\nu_o/4\pi)\phi(\nu)n_1 A_{10} dV d\Omega d\nu dt$. This implies

$$\boxed{j_\nu = (h\nu_o/4\pi)\phi(\nu)n_1 A_{10}} \tag{23}$$

- Absorption Coefficient α_ν
  Using similar arguments and from the second term of 7 we obtain

$$\alpha_\nu I_\nu dV d\Omega d\nu dt = (h\nu_o/4\pi)\phi(\nu)(n_0 B_{01} - n_1 B_{10}) I_\nu dV d\Omega d\nu dt$$

$$\boxed{\alpha_\nu = (h\nu_o/4\pi)\phi(\nu)(n_0 B_{01} - n_1 B_{10})} \tag{24}$$

Absorption Coefficient

$$\alpha_\nu = (h\nu_o/4\pi)\phi(\nu)n_0 B_{10}(B_{01}/B_{10} - n_1/n_0)$$

$$\alpha_\nu = \frac{h\nu_o}{4\pi}\phi(\nu)n_0\frac{c^2}{2h\nu_o^3}A_{10}\left(3 - 3e^{\frac{-T_*}{T_S}}\right)$$

$$\alpha_\nu \approx \frac{h\nu_o}{4\pi}\phi(\nu)n_0\frac{c^2}{2h\nu_o^3}A_{10}\left(3\frac{T_*}{T_S}\right) \tag{25}$$

Since $n_1 = n_0 e^{\frac{-T_*}{T_S}}$ and $T_* = 0.068K << T_S \sim T_\gamma = 2.73K$

$$n_o \approx n_{HI}/4$$

$$\implies \boxed{\alpha_\nu \approx \frac{3c^2 A_{10}}{32\pi\nu_0^2} n_{HI}\left(\frac{T_*}{T_S}\right)\phi(\nu)} \tag{26}$$

Optical Depth

$$\tau_\nu = \int \alpha_\nu dl$$

$$\tau_\nu = \int \frac{3c^2 A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \phi(\nu) dl$$

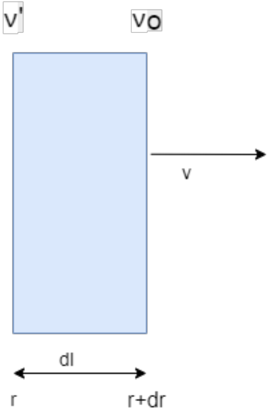
$$\tau_\nu = \frac{3c^2 A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \int_{\text{length of cloud}} \phi(\nu) dl$$

$$\text{Using} \int \phi(\nu) d\nu = 1$$

$$\boxed{\phi(\nu) \approx \frac{1}{\Delta\nu}}$$

$$\tau_\nu = \frac{3c^2 A_{10}}{32\pi\nu_0^2} n_{HI} \left(\frac{T_*}{T_S}\right) \frac{\Delta l}{\Delta\nu} \tag{27}$$

Optical Depth



$$\nu' = \nu_o(1 - v/c)$$

$$\Delta\nu = \nu_o(v/c)$$

$$\Delta l = a\Delta r$$

$$v = \frac{\Delta l}{\Delta t} = \dot{a}\Delta r + \frac{\partial v}{\partial r}\Delta r$$

$$\frac{\Delta l}{\Delta \nu} = \frac{a\Delta r}{\nu_o v/c} = \frac{c}{\nu_o}\frac{a\Delta r}{\dot{a}\Delta r + \frac{\partial v}{\partial r}\Delta r}$$

$$= \frac{c}{\nu_o H}\left(1 + \frac{1}{aH}\frac{\partial v}{\partial r}\right)^{-1}$$

$$\boxed{\frac{\Delta l}{\Delta \nu} \approx \frac{c}{\nu_o H}\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right)} \qquad (28)$$

$$\tau = \frac{3c^3 A_{10}}{32\pi\nu_0^3 H} n_H \left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \qquad (29)$$

Optical Depth

$$\tau = \frac{3c^3 A_{10}}{32\pi\nu_0^3 H} n_{HI} \left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right)$$

$$\tau = (1+z)\hat{T}(z)\frac{\rho_{HI}}{\rho_H}\left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \tag{30}$$

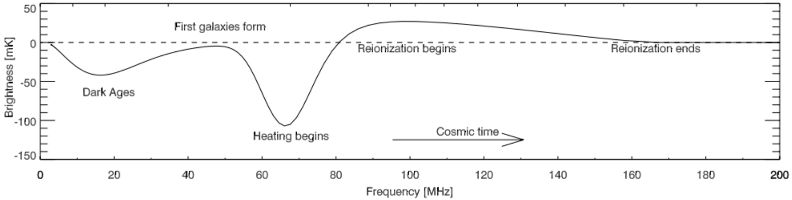
where we combine all prefactors in $\hat{T}(z)$

$$\hat{T}(z) = 4mK(1+z)^2\frac{\Omega_b h^2}{0.02}\frac{0.7}{h}\frac{H_o}{H_a} \tag{31}$$

From (13) the differential brightness temperature is

$$\delta T_B = \left(1 - \frac{T_B}{T_S}\right)\hat{T}(z)\frac{\rho_{HI}}{\rho_H}\left(\frac{T_*}{T_S}\right)\left(1 - \frac{1}{aH}\frac{\partial v}{\partial r}\right) \tag{32}$$

Global 21cm Signal



- The global 21cm signal is the angle averaged version of the the 21cm signal coming from all directions of the sky. Due to this angle averaging the $\frac{\partial v}{\partial r}$ in 32 becomes zero and we obtain

$$\delta T_B = \left(1 - \frac{T_B}{T_S}\right) \hat{T}(z) \frac{\rho_{HI}}{\rho_H} \left(\frac{T_*}{T_S}\right) \tag{33}$$

Three cases arise

1. $T_S > T_B \implies \delta T_B > 0$, Emission Signal
2. $T_S < T_B \implies \delta T_B < 0$, Absorption Signal
3. $T_S = T_B \implies \delta T_B = 0$, No Signal

Summary

- Neutral-H clouds leave an imprint on background radio signals (like CMB or Quasars) in the form of temperature fluctuations
- These temperature fluctuations depend on the spin temperature of the cloud. The spin temperature couples to the brightness temperature of background radio source or Kinetic Temperature of H gas depending on the neutral H fraction
- As our universe evolves, it passes through through certain epochs where the values of the spin temperature change quite drastically. This leads to a significant (measurable) variation in Brightness Temperature fluctuations across various redshifts

References

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- "Radiative Processes in Astrophysics" Rybicki and Lightman, 1985
- 21cm Physics and Cosmology Lectures by Somnath Bharadwaj at ICTS